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DIRECT BIT DETECTION RECEIVER PERFORMANCE ANALYSES FOR 8-DPSK AND 16-DPSK MODULATED SIGNALS OPERATING WITH IMPROPER CARRIER PHASE SYNCHRONIZATION

by

Mehmet Sevki Sekerefeli

December 1987

Thesis Advisor Daniel C. Bukofzer

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Direct Bit Detection Receiver Performance Analyses for 8-DPSK and 16-DPSK Modulated Signals Operating With Improper Carrier Phase Synchronization

by

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ABSTRACT

The application of Direct Bit Detection methods are analyzed and evaluated in digital communication systems employing Differential Phase Shift Keyed (DPSK) modulation. Assuming an additive white Gaussian noise interference model, and direct bit detection methods used in the receiver, 8-DPSK and 16-DPSK communication systems are considered and their performances evaluated in terms of the delivered bit error rate as a function of signal-to-noise ratio. The advantages and disadvantages of Direct Bit Detection Receiver (DBDR) systems used in conjuction with differential phase encoding are determined, with specific application to 8-DPSK and 16-DPSK modulated signals. The effect of improper receiver carrier phase synchronization is considered in detail, and resulting performance degradations are evaluated. Numerical results show that for receiver phase errors of more than a few degrees, severe performance degradations result for both DPSK modulation schemes, unless a complete phase reversal (i.e.,180°) takes place.

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I. INTRODUCTION

M-ary differential phase shift keyed (M-DPSK) modulation is a widely used technique in digital communication applications. Bandwidth efficiency, relatively good noise immunity, constant signal envelope, and simplicity of implementations, make this scheme particularly attractive for use over satellite, terrestrial radio and voiceband telephone channels. While system analyses pertaining to the performance of M-DPSK receivers abound in the literature, treatment is usually restricted to the case where signal transmission takes place over an additive white Gaussian noise channel. [Ref. 1]

Since the digital information transmission is based on carriers modulated by the symbol waveforms, the symbol error probabilities which quantify receiver performance can often be computed directly as demonstrated by the many derivations of such probabilities found in the literature (see [Ref. 2: pp. 228-234] and [Ref. 3: pp. 200-212] for example). However, from the point of view of the recipient of the digital information, it is the Bit Error Ratio (BER) that becomes important in digital communication applications. While results on the BER performance of M-ary modulation receivers is far more limited, closed form expressions for the BER of M-PSK receivers have been obtained by Lee [Ref. 4], with similar results independently derived by Tan [Ref. 5].

This thesis is devoted to analyzing the application of direct bit detection methods which allow the recovery of bits in the received symbol individually, in differential PSK digital communication systems. For analysis purposes, 8-DPSK and 16-DPSK systems are considered and their performances evaluated. The particular question to be answered is whether for 8-DPSK and 16-DPSK modulated signals, direct bit detection methods provide comparable performance to conventional phase detection receivers, especially under conditions of improper receiver carrier phase synchronization.

In this introductory chapter, background information on M-PSK communication systems is given. Known optimum receiver structures are presented, and Direct Bit Detection Receivers (DBDR) are introduced. In Chapters II and III a presentation of the transmitter and receiver structures respectively is given along with their operational properties, and the logic implemented in the DBDR for 8-DPSK and 16-DPSK modulated signals is discussed in detail. Chapter IV includes the noise performance analyses of 8-PSK and 8-DPSK receivers, and Chapter V presents similar receiver analyses for 16-PSK and 16-DPSK. In both of these chapters, receiver local oscillator phase errors have been incorporated so as to be able to account for degrading effects due to improper carrier phase synchronization. The obtained mathematical expressions of receiver performance are evaluated on a computer and graphically presented as plots of bit error ratio as a function of signal to noise ratio (SNR) and phase errors.

A. BACKGROUND

Direct bit detection methods utilize two signal processing channels in the receiver in order to recover the digital information without phase angle measurements. Moreover, these methods exhibit optimum performance in a BER sense when carrier synchronization is completely achieved. Because of the attractiveness and simplicity of DBDRs, some studies have been carried out in this area, and a modified two-channel receiver for 8-PSK has been built and its noise performance measured by Thompson [Ref. 6]. Furthermore, noise performance evaluations of DBDR for 8-PSK have been presented by Myers and Bukofzer [Ref. 7], while similar analyses for the 16-PSK case can be found in Reference 8. Advantages of direct bit detection methods are well documented in these referenced works, in which all analyses have been done assuming no receiver local oscillator phase error, Gray Code bit to symbol mapping and an additive white Gaussian noise interference model. We present here 8-PSK, 8-DPSK, 16-PSK and 16-DPSK communication systems under similar assumptions, except now the receiver has an assumed known local oscillator phase error.

B. THEORY OF M-PSK COMMUNICATION SYSTEMS

1. Representation of M-PSK Signals

For M-PSK modulation with equal signal energies, a convenient representation [Refs. 3: p.p. 192-205] of the signal set is given by

$$s_i(t) = \sqrt{2E_s/T_s}\cos[2\pi f_0 t + 2\pi(i-1)/M], \quad i=1,2,\ldots,M \quad , \quad 0 \le t \le T_s \qquad (1.1)$$

$$E_s = \int_0^{T_s} s_i^2(t)\,dt \quad , \qquad i=1,2,..,8 \qquad (1.2)$$

T_s = symbol duration

f_0 = carrier frequency (Hz.)

for convenience $f_o >> 1/T_s$ is assumed.

A suitable orthonormal signal set for the representation of the above signals is

$$\varphi_1(t) = \sqrt{2/T_s}\cos(2\pi f_o t)$$
$$\varphi_2(t) = \sqrt{2/T_s}\sin(2\pi f_o t) \quad , \quad 0 \le t \le T_s \tag{1.3}$$

so that by using trigonometric identities, $s_i(t)$ can be expanded in terms of $\varphi_1(t)$ and $\varphi_2(t)$. That is,

$$s_i(t) = \varphi_1(t)\sqrt{2E_s}\cos[\,2\pi(i-1)/M\,] - \varphi_2(t)\sqrt{2E_s}\sin[\,2\pi(i-1)/M\,] \tag{1.4}$$

where $i = 1,2,\ldots\ldots,M$.

A plot of the signal constellation for M-PSK is shown in Figure 1.1 for various values of M, where the coordinates of the i^{th} signal vector representing the signal $s_i(t)$ are

$$\{\sqrt{2E_s}\ \cos[2\pi(i-1)/M]\}\ ,-\ \{\sqrt{2E_s}\ \sin[2\pi(i-1)/M]\} \tag{1.5}$$

For M-DPSK, the signal representations are exactly same as those already presented, except that the digital information is transmitted in terms of phase differences between consecutive signal transmissions, rather than absolute signal phases.

2. Gray Coding for M-PSK Signals

Before proceeding with the Gray coding for M-PSK signals, we introduce some definitions which will henceforth be used

1. *Symbol* : Transmitted or received signal in sinusoidal form ($s_i(t)$ where $i = 1,2,\ldots M$) which is related to the symbol state.
2. *Symbol state* : The binary digit assignment to the symbols which always consists of a block of $\log_2 M$ bits in length($\underline{S}_i$ where $i = 1,2,\ldots,M$.)
3. *Data state* : Block of $\log_2 M$ bits corresponding to the data that must be delivered to the intended user, and which is processed at the encoder input to determine the symbol to be transmitted. ($\underline{D}_i$ where $i = 1,2,\ldots.,M$.)

The relationship between symbol error probability and bit error probability in the corresponding $\log_2 M$ bit groups depends upon the assignment of bits to the symbol states. The preferred bit to symbol state mapping in most cases is the Gray Code, in which the symbol states which correspond to adjacent phases differ in only one bit position. This is because in the coherent demodulation process the most likely error

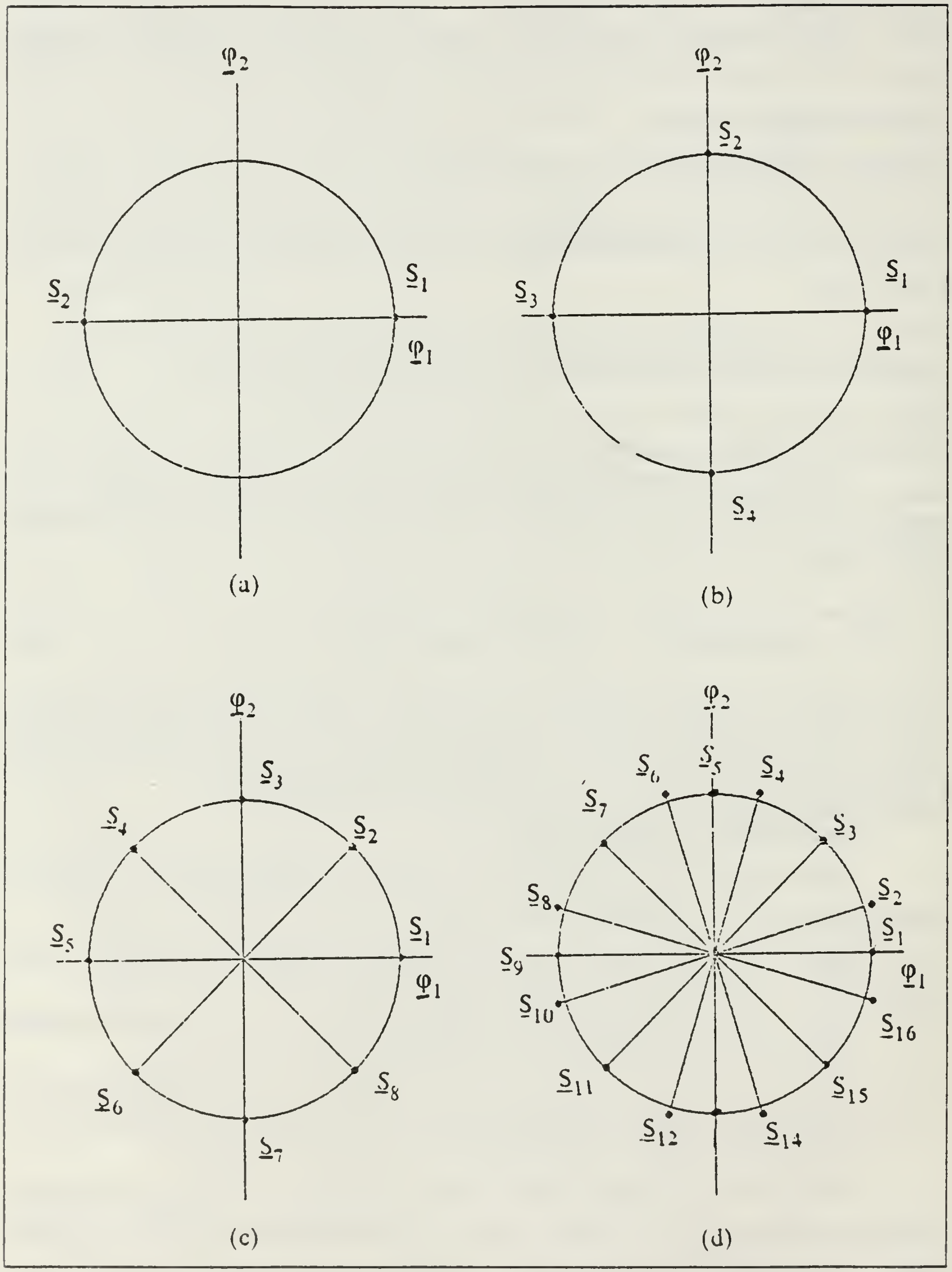

Figure 1.1 Signal Constellations for (a) BPSK (b) QPSK (M = 4) (c) 8-PSK (d) 16-PSK.

involves mistaking a correct symbol with its nearest neighbor and so such symbol errors translate into single bit errors.

In Direct Bit Detection Receivers (DBDR), Gray code assignment to the symbol states are selected for the above given reason. In this thesis, a modified Gray code is used, which for comparison purposes is shown along with the conventional Gray Code in Table 1.

TABLE 1
COMPARISON OF CONVENTIONAL AND MODIFIED GRAY CODING SCHEMES

Symbol States	Gray Coding		Modified Gray Coding	
	8-PSK	16-PSK	8-PSK	16-PSK
$\underline{S}_1$	000	0000	111	1111
$\underline{S}_2$	001	0001	101	1101
$\underline{S}_3$	011	0011	001	1001
$\underline{S}_4$	010	0010	011	1011
$\underline{S}_5$	110	0110	010	0011
$\underline{S}_6$	111	0111	000	0001
$\underline{S}_7$	101	0101	100	0101
$\underline{S}_8$	100	0100	110	0111
$\underline{S}_9$		1100		0110
$\underline{S}_{10}$		1101		0100
$\underline{S}_{11}$		1111		0000
$\underline{S}_{12}$		1110		0010
$\underline{S}_{13}$		1010		1010
$\underline{S}_{14}$		1011		1000
$\underline{S}_{15}$		1001		1100
$\underline{S}_{16}$		1000		1110

Using the modified Gray Code on Table 1, the vectors representing the symbols with components along the φ_1 and φ_2 axes have been shown (see Figure 1.2) such that r_1 is the symbol component along the φ_1 axis and r_2 is the symbol component along the φ_2 axis. Therefore for M = 8, (see Figure 1.2.a) in order to recover Most Significant Bit (LB) of the transmitted symbols correctly, $r_1 > 0$ whenever the symbol states $\underline{S}_1$, $\underline{S}_2$, $\underline{S}_7$ and $\underline{S}_8$ are transmitted and $r_1 < 0$ whenever the symbol states $\underline{S}_3$, $\underline{S}_4$, $\underline{S}_5$ and $\underline{S}_6$ are transmitted. The Middle Bit (MB) of the transmitted symbols can be recovered correctly if $(r_1)^2 - (r_2)^2 > 0$ whenever the symbol states $\underline{S}_1$, $\underline{S}_4$, $\underline{S}_5$ and $\underline{S}_8$ are transmitted and $(r_1)^2 - (r_2)^2 < 0$ whenever the symbol states $\underline{S}_2$, $\underline{S}_3$, $\underline{S}_6$ and $\underline{S}_7$ are transmitted. The Least Significant Bit (RB) is recovered correctly if $r_2 > 0$ whenever the symbol states $\underline{S}_1$, $\underline{S}_2$, $\underline{S}_3$, and $\underline{S}_4$ are transmitted and $r_2 < 0$ whenever the symbol states $\underline{S}_5$, $\underline{S}_6$, $\underline{S}_7$, and $\underline{S}_8$ are transmitted.

Similar logic has been set up for the M = 16 case (see Figure 1.2.b) but now each symbol is related to 4 bits in a symbol state. Therefore for this case, 4 bits must be recovered from each received symbol, where the bits in a symbol state are labeled Most Significant Bit (LB), Middle Bit Right (MBR), Middle Bit Left (MBL), and Least Significant Bit (RB). For example from Figure 1.2.b, $\underline{S}_7$ is 0101, corresponding to LB = 0, MBL = 1, MBR = 0 and RB = 1. In order to recover the LB correctly, it is necessary that $r_1 > 0$ for transmitted symbol states $\underline{S}_1$, $\underline{S}_2$, $\underline{S}_3$, $\underline{S}_4$, $\underline{S}_{13}$, $\underline{S}_{14}$, $\underline{S}_{15}$, and $\underline{S}_{16}$ and that $r_1 < 0$ for transmitted symbol states $\underline{S}_5$, $\underline{S}_6$, $\underline{S}_7$, $\underline{S}_8$, $\underline{S}_9$, $\underline{S}_{10}$, $\underline{S}_{11}$, and $\underline{S}_{12}$. If $r_2 > 0$ whenever the symbol states $\underline{S}_1$, $\underline{S}_2$, $\underline{S}_3$, $\underline{S}_4$, $\underline{S}_5$, $\underline{S}_6$, $\underline{S}_7$, and $\underline{S}_8$ are transmitted and $r_2 < 0$ whenever the symbol states $\underline{S}_9$, $\underline{S}_{10}$, $\underline{S}_{11}$, $\underline{S}_{12}$, $\underline{S}_{13}$, $\underline{S}_{14}$, $\underline{S}_{15}$, and $\underline{S}_{16}$ are transmitted, the RB will be recovered correctly. If $(r_1)^2 > (r_2)^2$ for transmission of symbol states $\underline{S}_1$, $\underline{S}_2$, $\underline{S}_7$, $\underline{S}_8$, $\underline{S}_9$, $\underline{S}_{10}$, $\underline{S}_{15}$, and $\underline{S}_{16}$, and $(r_1)^2 < (r_2)^2$ for transmission of symbol states $\underline{S}_3$, $\underline{S}_4$, $\underline{S}_5$, $\underline{S}_6$, $\underline{S}_{11}$, $\underline{S}_{12}$, $\underline{S}_{13}$, and $\underline{S}_{14}$, the MBL will be recovered correctly. Finally, if $[\,(r_1)^2 - (r_2)^2\,]^2 - (2\,r_1r_2)^2 > 0$ upon transmission of the symbol states $\underline{S}_1$, $\underline{S}_4$, $\underline{S}_5$, $\underline{S}_8$, $\underline{S}_9$, $\underline{S}_{12}$, $\underline{S}_{13}$, and $\underline{S}_{16}$, and $[\,(r_1)^2 - (r_2)^2\,]^2 - (2\,r_1r_2)^2 < 0$ upon transmission of the symbol states on the states $\underline{S}_2$, $\underline{S}_3$, $\underline{S}_6$, $\underline{S}_7$, $\underline{S}_{10}$, $\underline{S}_{11}$, $\underline{S}_{14}$, and $\underline{S}_{15}$, then the MBR will be recovered correctly.

In subsequent chapters, the receiver logic and its performance will be explained in detail. Additionally, application of this methodology will be presented in so far as differential PSK modulation is concerned. Before doing so however, a summary of known results on the performance of conventional and DBDR's for M-PSK and M-DPSK is presented next.

3. Known Results on the Performance of M-PSK and M-DPSK Receivers

In this section, known results on the receiver performance for M-PSK and DPSK modulated signals are presented for two cases, namely conventional receivers and Direct Bit Detection Receivers (DBDR).

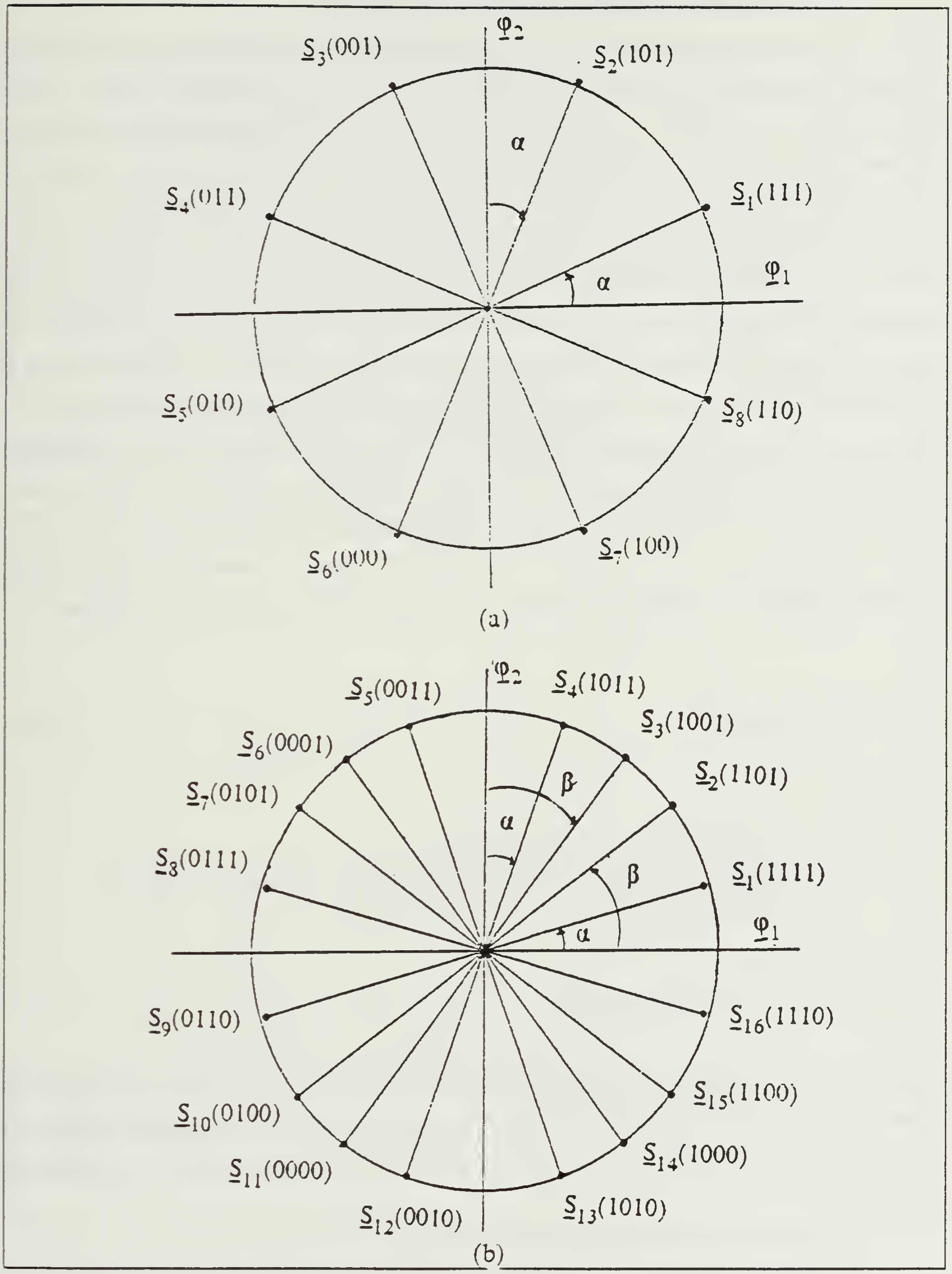

Figure 1.2 Symbol State Assignments for (a) 8-PSK (b) 16-PSK.

a. Conventional M-PSK and M-DPSK Receivers

Coherent demodulation of M-PSK implies in principle the use of M signal processing channels in the receiver, as shown in Figure 1.3 (from Ha [Ref. 10]) where different forms of the optimum receiver structure of M-PSK signaling have been given. Performance analysis results for optimum receivers are well-documented in terms of symbol error rate as a function of signal to noise ratio [Refs. 2,9], while more recent analyses have focused on the receiver probability of bit error (see [Ref. 4] and [Ref. 3: p.p. 204-207] for example). Symbol error probabilities can be computed from knowledge of the channel and signal characteristics. However for transmission of binary data, when analyzing system performances as a function of different levels of modulation, the bit error probability rather than the symbol error probability is of interest, as previously explained. In Prabhu [Ref. 1: p.198], the bit error probability has been presented only for the case of M-ary orthogonal signal sets. However, since the M-PSK signal set is not orthogonal, those results are not applicable here. The closed form expression for the BER of M-PSK has been determined, when a Gray Code bit mapping is used [Ref.2: p.198]. The results can be summarized as follows

$$P_b(M) = \sum_{k=1}^{n} (k/n)\, P_k \tag{1.6}$$

where

M = Number of distinct signal waveform,

P_k = Probability of k bits in error in a received n bit data block [Ref. 5: pp.24-32],

$n = \log_2 M$,

$P_b(M)$ = Bit error probability.

A derivation of Equation 1.6 and its numerical evaluation (see Figure 1.4) has been carried out by Tan [Ref. 5: p.33]. For M-PSK signalling, receiver performance analyses in terms of bit error rate have also been presented by Lee [Ref. 4: p.491], and plotted in Figure 1.5.

b. Direct Bit Detection Receivers (DBDR's)

DBDR structures for M-PSK have been derived for various values of M [Refs. 6,7,8]. As previously indicated, this method is particularly well suited for M-PSK communication systems. The DBDR structures for 8-PSK (see Figure 1.6) and 16-PSK

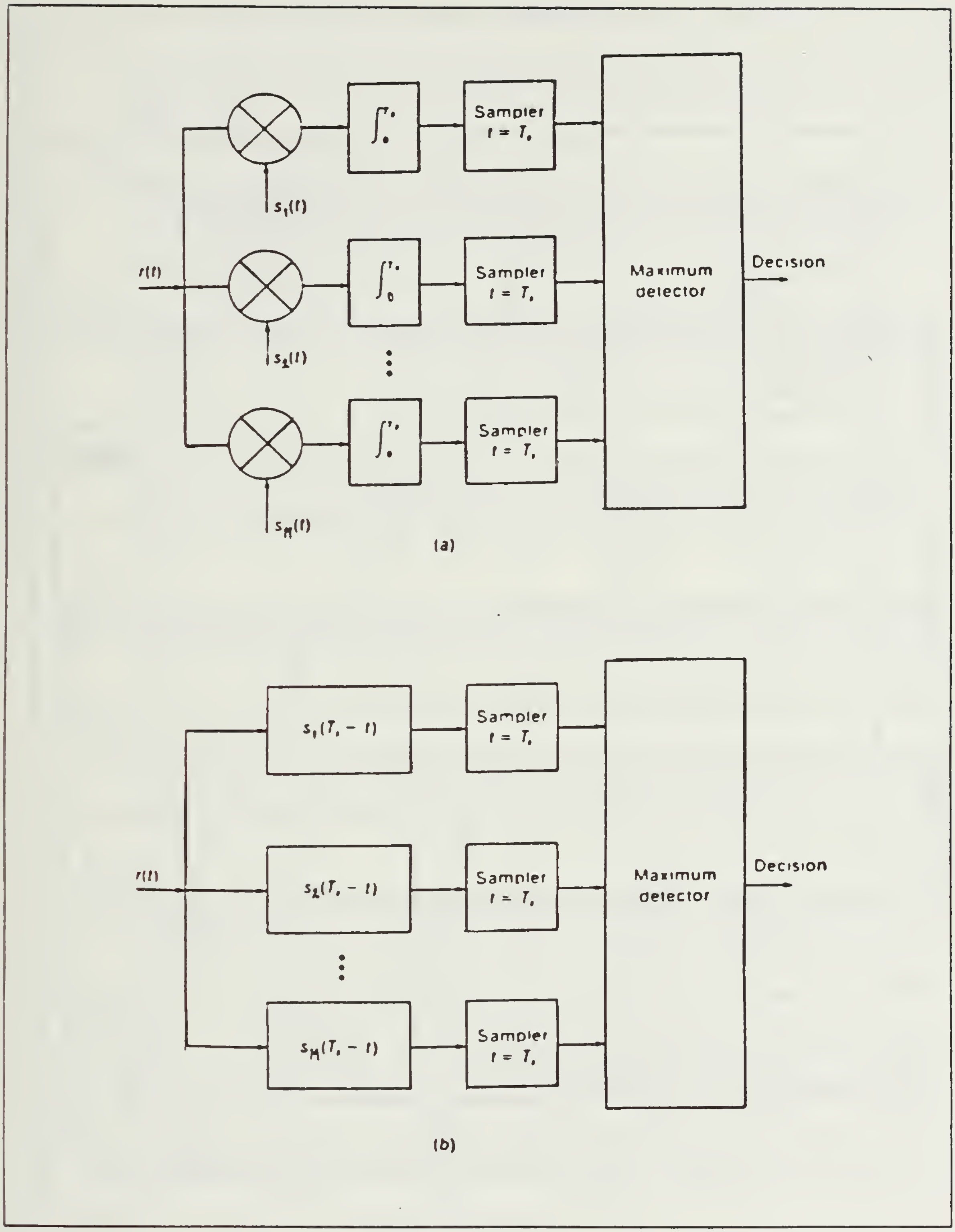

Figure 1.3 Block Diagram of Optimum Receiver (a) Correlator Realization (b) Matched Filter Realization (from Ha [Ref. 10]).

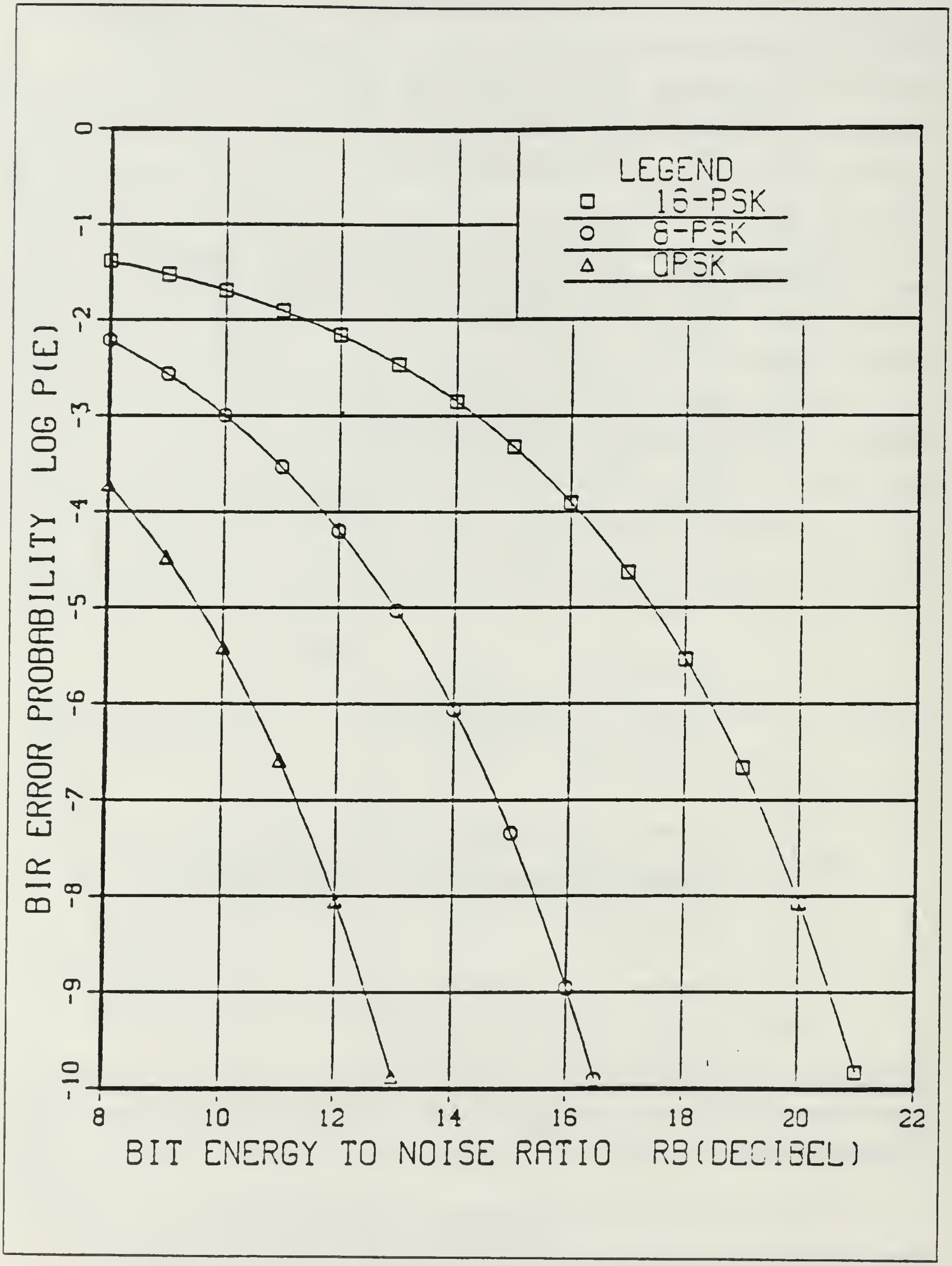

Figure 1.4 Bit Error Probabilities Versus E_b/N_0 for 16-PSK, 8-PSK and QPSK [Ref. 5 p.31].

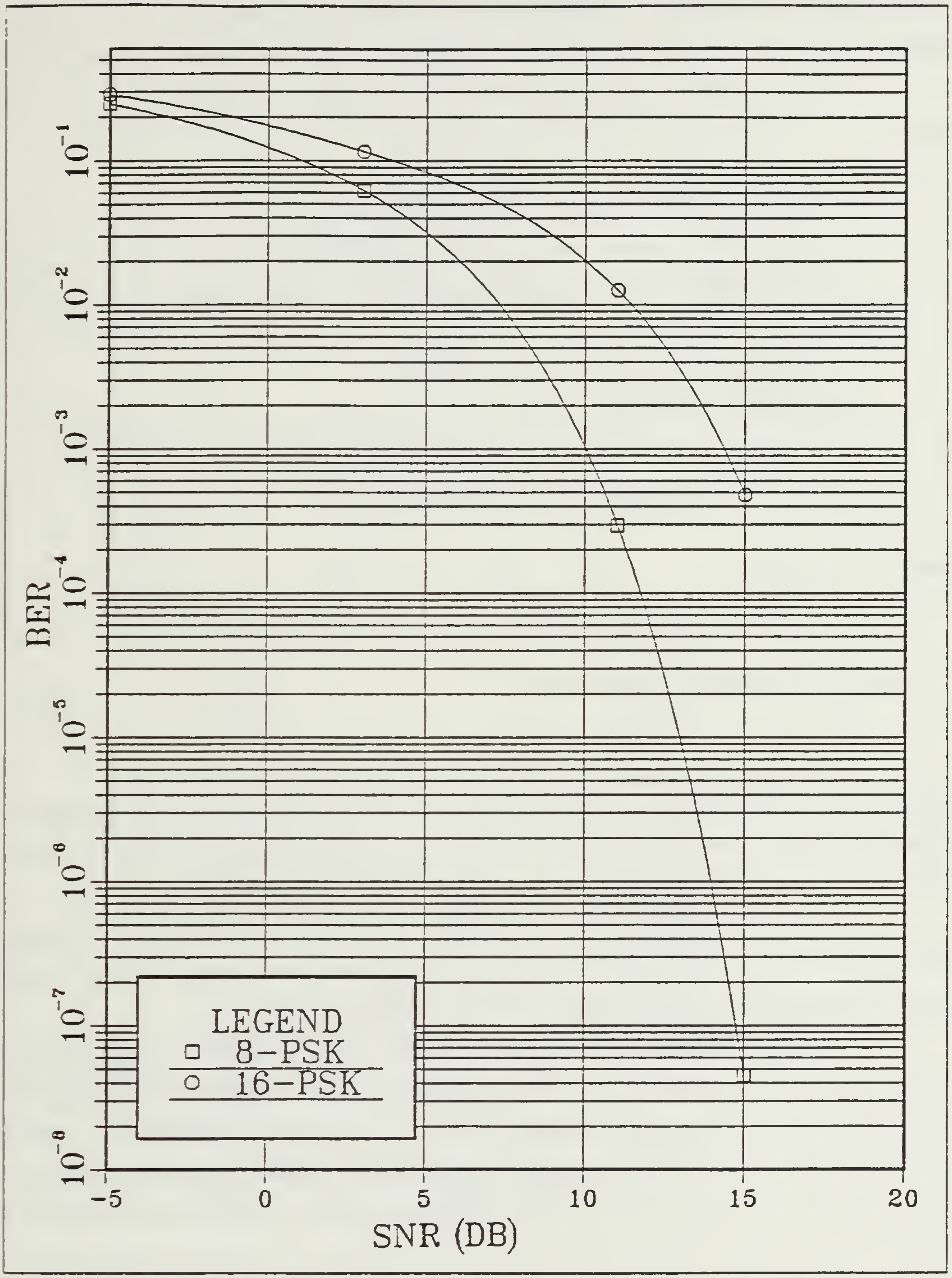

Figure 1.5 Probability of Bit error Versus E_b/N_0 [Ref.4].

(see Figure 1.8) have been presented and their performances have been derived in References 6,7,8. The BER is quantified by the probabilities of a bit error, denoted PE.

For 8-PSK, PE has been derived by Myers and Bukofzer [Ref. 7: p.13] where

$$\begin{aligned} PE = 1/3 \{ & Q[2\sqrt{SNR}\,\cos\alpha] + Q[2\sqrt{SNR}\,\sin\alpha] + \\ & + Q[2\sqrt{SNR}\,(\cos\alpha + \sin\alpha)] + Q[2\sqrt{SNR}\,(\cos\alpha - \sin\alpha)] - \\ & - 2\, Q[2\sqrt{SNR}\,(\cos\alpha - \sin\alpha)] \cdot Q[2\sqrt{SNR}\,(\cos\alpha + \sin\alpha)] \} \end{aligned} \qquad (1.7)$$

where

$SNR = E_s/2N_0 = 3E_b/2N_0$
E_s = Symbol energy
N_0 = Noise power spectral density level (one sided)
α = Basic signal phase angle (see Figure 1.2).

and

$$Q(x) = \int_x^{\infty} [1/\sqrt{2\pi}]\ \exp(-u^2/2)\ du \qquad (1.8)$$

The derivation of Equation 1.7 assumes an AWGN channel, Gray coded symbols and equal a priori bit probabilities (i.e. , 1/2). Receiver performance in BER sense is plotted in Figure 1.7 for various values of α as a function of SNR.

For the 16-PSK case, probability of bit error is derived by Bukofzer [Ref. 8] and the resulting receiver performance is plotted in Figure 1.9 for various values of the angles α and β, where β is shown on Figure 1.2.b.

In both analyses, it has been assumed that the receiver's local oscillator does not have any phase error. In this research, similar analyses to those conducted in References 7 and 8 have been carried out by considering the effect of a receiver local oscillator phase error for 8-PSK and 16-PSK cases, as well as the noise performance of 8-DPSK and 16-DPSK DBDR, presented as BER as a function of SNR.

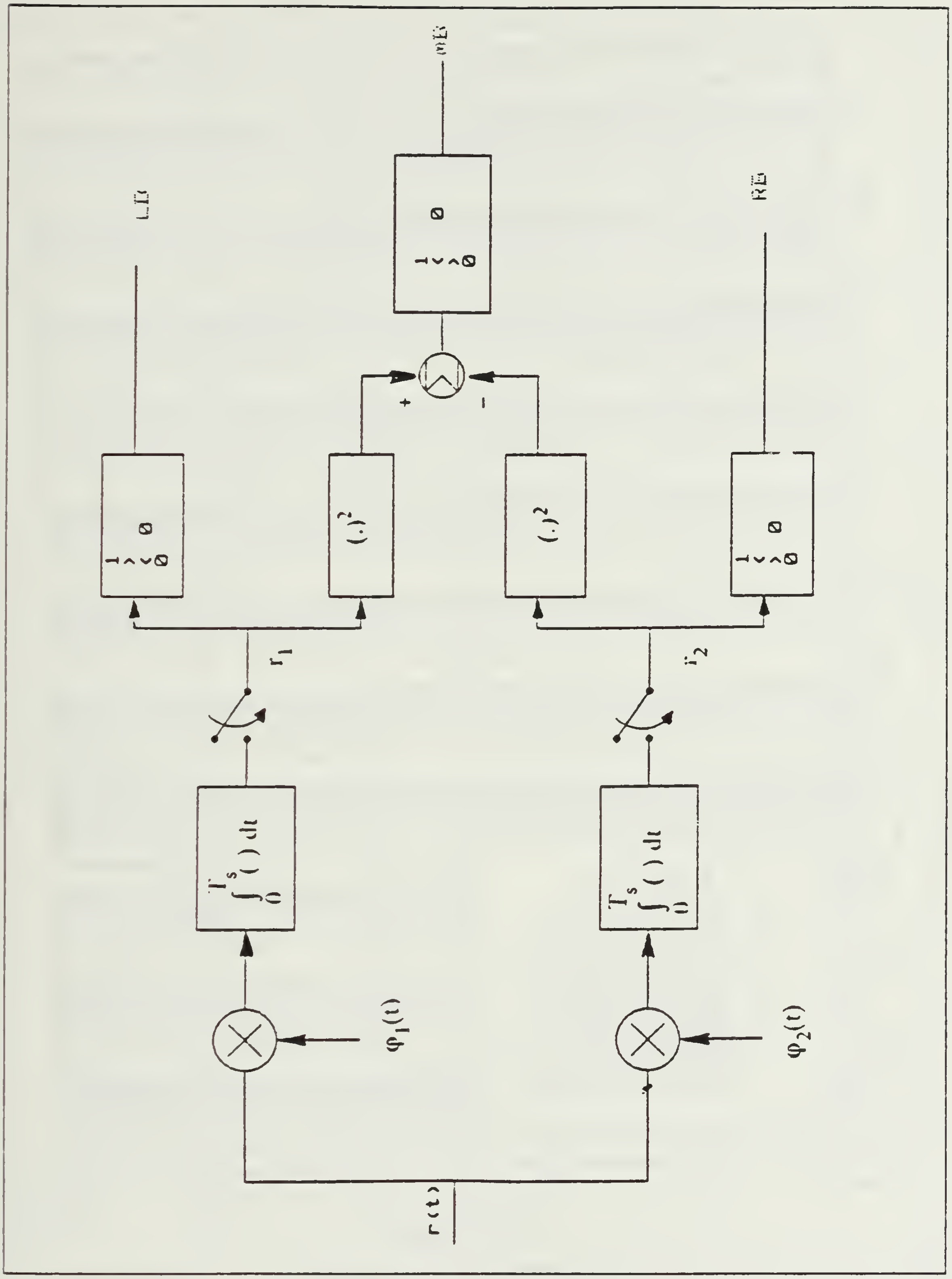

Figure 1.6 Block Diagram for 8-PSK DBDR.

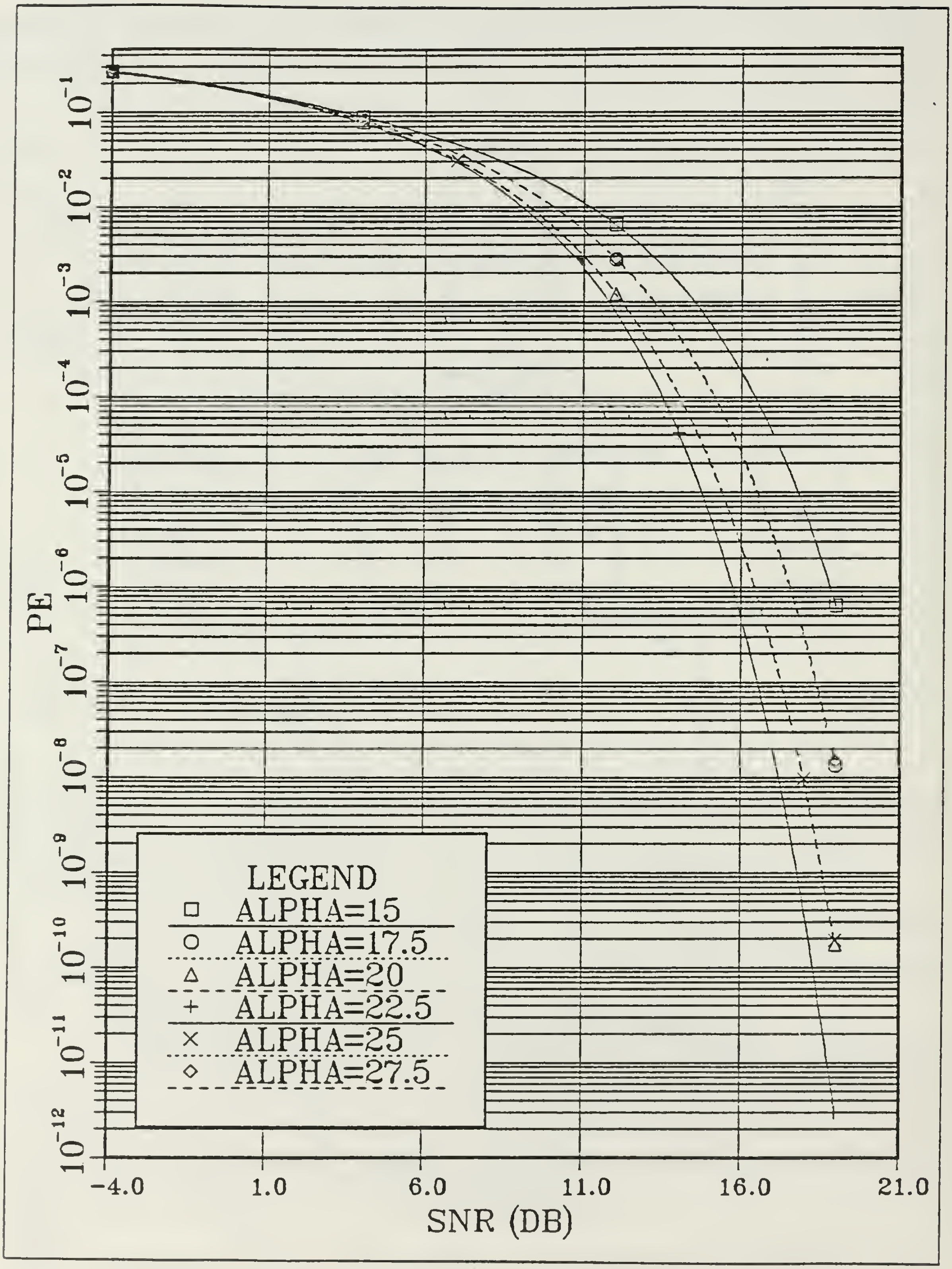

Figure 1.7 PE Versus SNR (dB) for 8-PSK for Various Values of α.

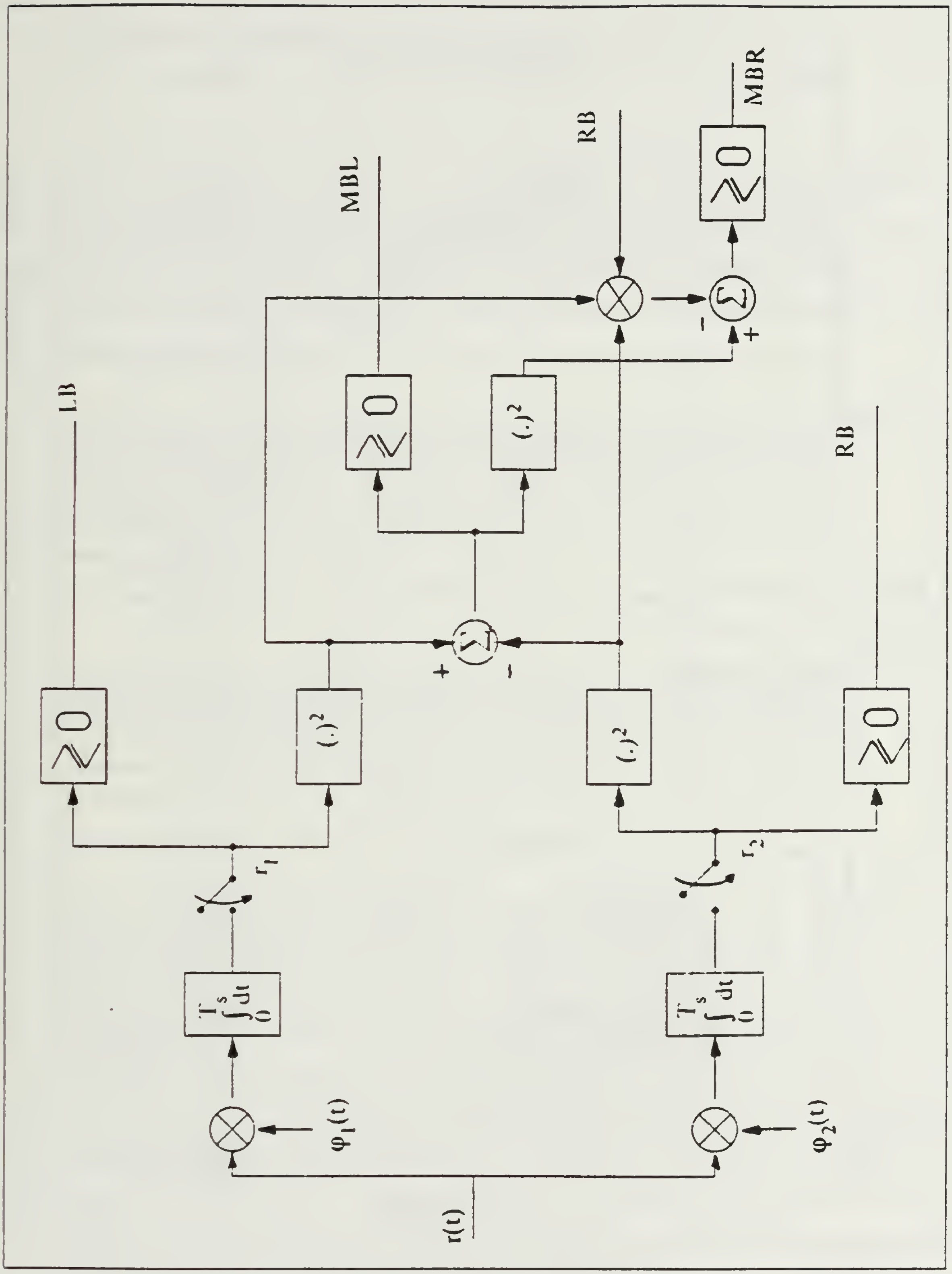

Figure 1.8 Block Diagram for 16-PSK DBDR.

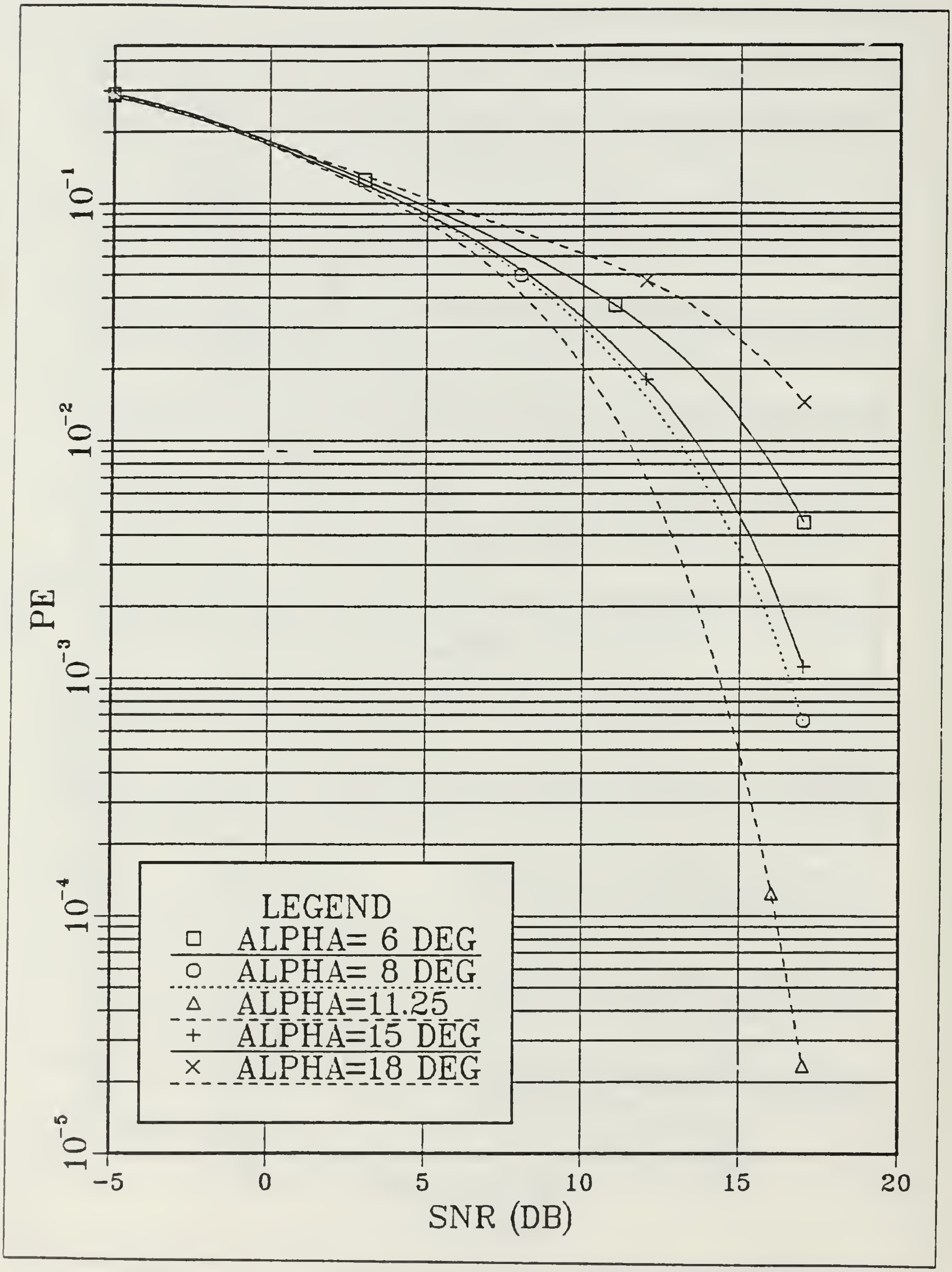

Figure 1.9 PE Versus SNR (dB) for Various α Values for 16-PSK DBDR.

II. TRANSMITTER

The generation of differential PSK signals can be performed by pre-processing the data bits. This chapter is devoted to analyzing the M-DPSK transmitter structure. One general form of the transmitter in block diagram form for DPSK communication systems is shown in Figure 2.1. Explanation of the general elements in Figure 2.1 will be included along with the derivation of the transmitter structure for 8-DPSK and 16-DPSK modulated signals.

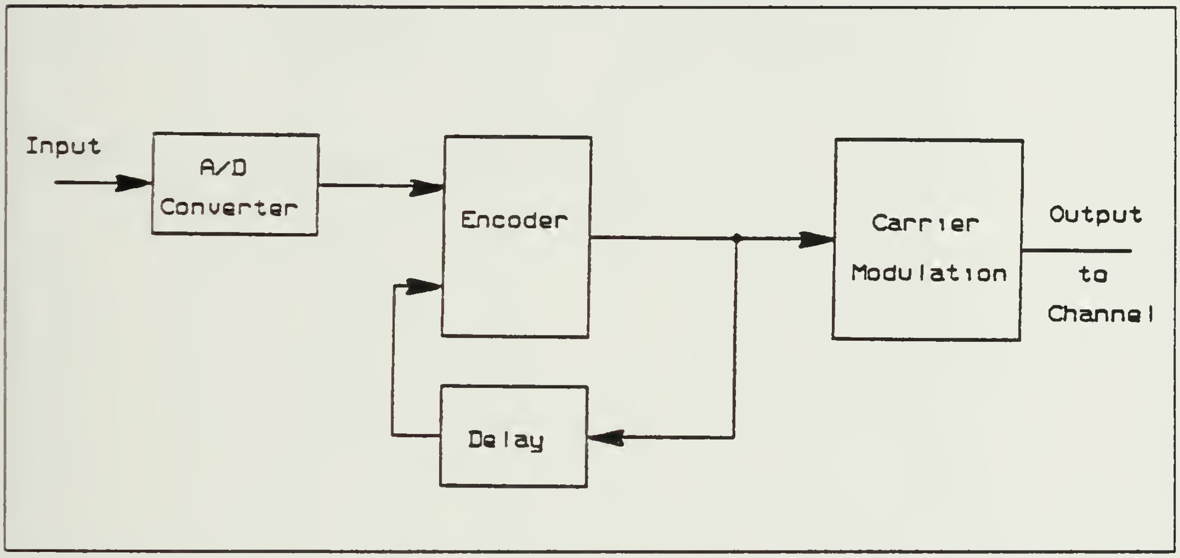

Figure 2.1 DPSK Transmitter Block Diagram.

A. TRANSMITTER STRUCTURE FOR 8-DPSK AND 16-DPSK MODULATED SIGNALS

The main task in generating the DPSK transmitter involves the design of the encoder, which depends on the algorithm used to relate data states to the symbol states prior to transmission as a sinusoidal waveform. Basically, the transmitted symbol is the quantized equivalent of the angle between the successively transmitted symbols. For reasons previously explained, a Gray coding scheme is utilized in the symbol state to symbol mapping for the two special cases of M-DPSK signalling considered here, namely $M = 8$ and $M = 16$.

The symbol differential phase relative to the data state to be transmitted is given on Table 2 for 8-DPSK and on Table 3 for 16-DPSK modulated signals. In both Tables, it is assumed that an equiphase arrangement for the transmitted symbols is used.

TABLE 2

GRAY CODE REPRESENTATION OF PHASES FOR 8-DPSK SIGNAL SET

LB	MB	RB	Differential Phase
0	0	0	0
0	0	1	α
0	1	1	2α
0	1	0	3α
1	1	0	4α
1	1	1	5α
1	0	1	6α
1	0	0	7α

As mentioned earlier, the main purpose of the encoder is to generate a mapping algorithm whereby the phase difference of consecutive symbol outputs is related to the input data in accordance with the information in Table 2 and Table 3. In order to achieve successive signal transmissions the following algorithm is implemented in the transmitter

1. Store the present data state ($\underline{D}_i$, $i=1,2...,M$) and the previous symbol state ($\underline{S}_i$,$i=1,2,3..,M$).
2. From the Table 2 or Table 3, find the corresponding differential phase value associated with the present data state as $N\alpha$, where N is an integer with values of $0 \leq N \leq M-1$.
3. Generate a symbol state, such that the corresponding symbol will have a phase angle $N\alpha$ degrees larger (in additive sense, modulo 2π) than the previously transmitted symbol.

TABLE 3

GRAY CODE REPRESENTATION OF PHASES FOR 16-DPSK SIGNAL SET

LB	MBL	MBR	RB	Differential Phase
0	0	0	0	0
0	0	0	1	α
0	0	1	1	2α
0	0	1	0	3α
0	1	1	0	4α
0	1	1	1	5α
0	1	0	1	6α
0	1	0	0	7α
1	1	0	0	8α
1	1	0	1	9α
1	1	1	1	10α
1	1	1	0	11α
1	0	1	0	12α
1	0	1	1	13α
1	0	0	1	14α
1	0	0	0	15α

For example, assuming the previous symbol state is $\underline{S}_7$ (0101) and present data state is $\underline{D}_9$ (0110) which corresponds to the differential phase value of 4α (see Table 3.). From Figure 1.2.b, proceed 4α degrres counterclockwise from $\underline{S}_7$, resulting in the symbol state is $\underline{S}_{14}$ (1111) will be the symbol output from the encoder.

This algorithm can be shown in matrix structure, in which the row entries and the column entries of the matrix are the symbol states and data states respectively, and the elements of the matrix are the symbol state which correspond to the appropriate phase difference in the transmitted symbol (see Figure 2.3 and Figure 2.2). For example, under the same assumptions given in the previous paragraph, by using $\underline{D}_9$ (0110) as a row entry and $\underline{S}_7$ as a column entry in the matrix in Figure 2.3, present symbol output from encoder is found as (1111) at the intersection of the row and column of the matrix.

	SYMBOL STATE							
	$\underline{S}_6$	$\underline{S}_7$	$\underline{S}_8$	$\underline{S}_1$	$\underline{S}_2$	$\underline{S}_3$	$\underline{S}_4$	$\underline{S}_5$
DATA STATE	000	100	110	111	101	001	011	010
$\underline{D}_6$= 000	000	100	110	111	101	001	011	010
$\underline{D}_7$= 100	100	110	111	101	001	011	010	000
$\underline{D}_8$= 110	110	111	101	001	011	010	000	100
$\underline{D}_1$= 111	111	101	001	011	010	000	100	110
$\underline{D}_2$= 101	101	001	011	010	000	100	110	111
$\underline{D}_3$= 001	001	011	010	000	100	110	111	101
$\underline{D}_4$= 011	011	010	000	100	110	111	101	001
$\underline{D}_5$= 010	010	000	100	110	111	101	001	011

Figure 2.2 Encoder Matrix for 8-DPSK Signals.

The hardware implementation of the encoder structure can be accomplished in several forms. A practical approach to building the encoder can be achieved by the use of EPROMs (Erasable Programable Read Only Memory). Once the truthe table the programming of the EPROM is a trivial task to accomplish. Demultiplexers can also be used to build the encoder circuitry; however, because such a device would be hard wired, it would not have the reprogrammability of the EPROM implementation.

In this research, the 8-DPSK and 16-PSK communication systems are evaluated using a computer. Appendix A includes the source file which is used to generate the possible outputs of an M-PSK and an M-DPSK communication system.

The carrier modulator uses the encoder outputs (symbol states) as an input, and generates the symbols (signals) suitable for transmission. The signal transmission involves sinusoidal voltages whose phase is related to the symbol states (encoder

	SYMBOL STATE														
	$\underline{S}_{11}$	S_6	$\underline{S}_5$	$\underline{S}_1$	$\underline{S}_9$	$\underline{S}_8$	$\underline{S}_7$	$\underline{S}_{10}$	$\underline{S}_{15}$	$\underline{S}_2$	$\underline{S}_1$	$\underline{S}_{16}$	$\underline{S}_{13}$	$\underline{S}_4$	$\underline{S}_3$
DATA STATE	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001
$\underline{D}_{11}$ = 0000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001
$\underline{D}_6$ = 0001	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000
$\underline{D}_5$ = 0011	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000
$\underline{D}_{12}$ = 0010	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001
$\underline{D}_9$ = 0110	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011
$\underline{D}_8$ = 0111	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010
$\underline{D}_7$ = 0101	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110
$\underline{D}_{10}$ = 0100	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111
$\underline{D}_{15}$ = 1100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101
$\underline{D}_2$ = 1101	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100
$\underline{D}_1$ = 1111	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100
$\underline{D}_{16}$ = 1110	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101
$\underline{D}_{13}$ = 1010	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111
$\underline{D}_4$ = 1011	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110
$\underline{D}_3$ = 1001	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010
$\underline{D}_{14}$ = 1000	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011

Figure 2.3 Encoder Matrix for 16-DPSK Signals.

outputs) and of carrier frequency suitable for the transmission channel. The signal generation can be achieved via a multiplexer, whose structure is shown in Figure 2.4. For this kind of implementation, (M-1) phase shift elements are needed with set phase shift values. Another approach would involve an implementation using a D/A converter along with a phase modulator which can be obtained with a VCO (see [Ref. 11: pp.269-271].

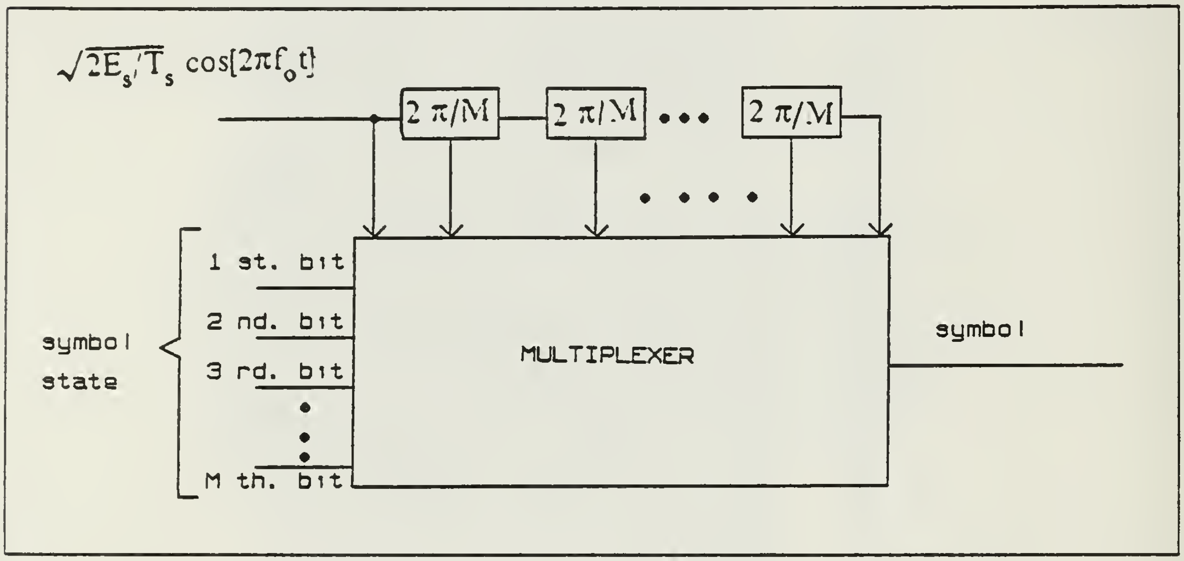

Figure 2.4 Carrier Modulator Structure for M-DPSK Signaling.

B. COMPUTER GENERATION OF THE TRANSMITTER OUTPUTS

For computer evaluation of the M-ary DPSK communication system , a FORTRAN program called BELIZ has been written (see Appendix A). The program generates all possible data state and symbol state combinations for M-DPSK, and produces the corresponding symbol states outputs. The program outputs are given in Appendix B in tabular form along with the transmitter matrix of Figure 2.3 and Figure 2.2, specified in decimal form.

III. RECEIVER

A. RECEIVER STRUCTURE FOR 8-DPSK AND 16-DPSK MODULATED SIGNALS

The introductory chapter presents an overview of the structure and performance of optimum receivers for M-ary signalling. The focus of this chapter is on DBDR's. As mentioned in Chapter II, the symbol state generation based on the data state and the previous symbol state is the most important task performed by the encoder. Therefore in the receiver's decoder, a similar algorithm must be performed in reversed sense, that is, from two successive symbol states, the corresponding data state must be estimated. A generic form of the receiver in block diagram structure is shown in Figure 3.1.

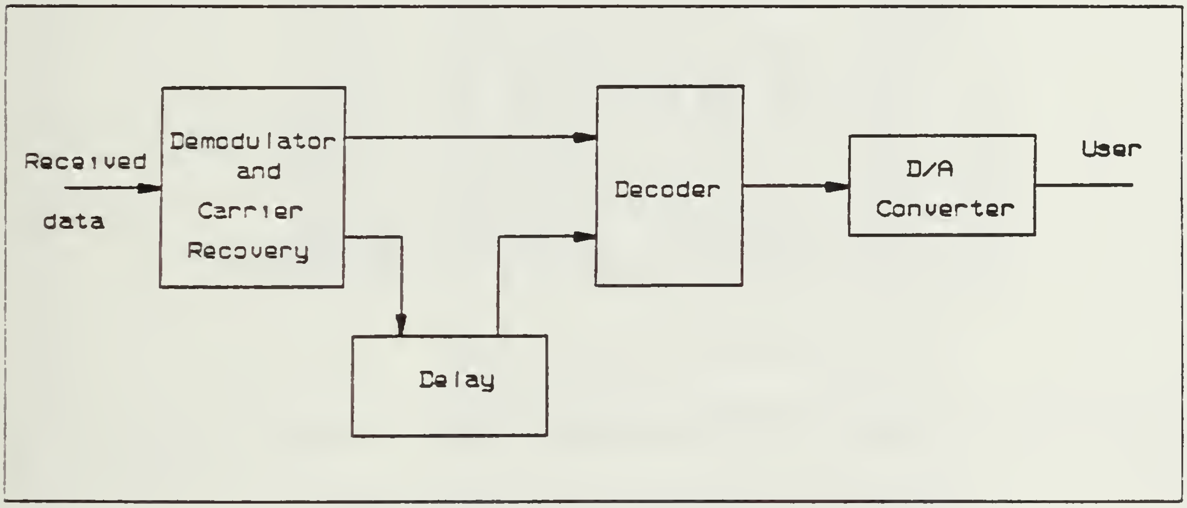

Figure 3.1 General receiver block diagram for DPSK signals.

The algorithm for recovering the transmitted data is as follows

1. Recover the symbol state from the received symbol using direct bit detection techniques.
2. Store the past symbol state $\underline{S}_i$,i = 1,2...,M.
3. Compare the present symbol state with the past symbol state to determine the incremental " phase angle " between the two.
4. From Table 2 or 3 find the corresponding data state that gave rise to the "phase angle" between the two consecutive symbol states.

For example, assuming the past symbol state is ($\underline{S}_6$) 0001 and the present symbol state is $\underline{S}_3$ (1001) from Figure 1.2.b, the angle between $\underline{S}_3$ and $\underline{S}_6$ is 3α. From Table 3., the corresponding Gray Code representation for 3α is found to be (0010) , therefore the recovered data state will be $\underline{D}_{12}$.

The decoder implementation can be depicted as a matrix structure in a manner similar to that shown for the transmitter encoder in Chapter II. For 8-DPSK and 16-DPSK modulated signals, the decoder matrices are given in Figure 3.2 and Figure 3.3 respectively. For example, assuming the same conditions as in the above paragraph, by using $\underline{S}_6$ and $\underline{S}_3$ as a column and row entries for the matrix given in Figure 3.3, the recovered data state from the decoder is determined as $\underline{D}_{12}$ (0010).

	SYMBOL STATE							
	$\underline{S}_6$	$\underline{S}_7$	$\underline{S}_8$	$\underline{S}_1$	$\underline{S}_2$	$\underline{S}_3$	$\underline{S}_4$	$\underline{S}_5$
SYMBOL STATE	000	001	011	010	110	111	101	100
$\underline{S}_6$ - 000	000	001	011	010	110	111	101	100
$\underline{S}_7$ - 100	100	000	001	011	010	110	111	101
$\underline{S}_8$ - 101	101	100	000	001	011	010	110	111
$\underline{S}_1$ - 111	111	101	100	000	001	011	010	110
$\underline{S}_2$ - 101	101	111	101	100	000	001	011	010
$\underline{S}_3$ - 010	010	101	111	101	100	000	001	011
$\underline{S}_4$ - 011	011	010	101	111	101	100	000	001
$\underline{S}_5$ - 001	001	011	010	101	111	101	100	000

Figure 3.2 Decoder matrix for 8-DPSK signals.

B. COMPUTER GENERATION OF RECEIVER OUTPUTS

Appendix C includes the results of the computer evaluation of the M-ary DPSK receiver. In addition to generating the decoder matrix in decimal form, the output for this program includes all possible combinations of the decoder outputs.

	SYMBOL STATE														
	$\underline{S}_{11}$	$\underline{S}_{6}$	$\underline{S}_{5}$	$\underline{S}_{1}$	$\underline{S}_{9}$	$\underline{S}_{8}$	$\underline{S}_{7}$	$\underline{S}_{10}$	$\underline{S}_{15}$	$\underline{S}_{2}$	$\underline{S}_{1}$	$\underline{S}_{16}$	$\underline{S}_{13}$	$\underline{S}_{4}$	$\underline{S}_{3}$
SYMBOL STATE	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001
$\underline{S}_{11}$ = 0000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001
$\underline{S}_{6}$ = 0001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011
$\underline{S}_{5}$ = 0011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010
$\underline{S}_{12}$ = 0010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110
$\underline{S}_{9}$ = 0110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111
$\underline{S}_{8}$ = 0111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101
$\underline{S}_{7}$ = 0101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100	1100
$\underline{S}_{10}$ = 0100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101	0100
$\underline{S}_{15}$ = 1100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111	0101
$\underline{S}_{2}$ = 1101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110	0111
$\underline{S}_{1}$ = 1111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010	0110
$\underline{S}_{16}$ = 1110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011	0010
$\underline{S}_{13}$ = 1010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001	0011
$\underline{S}_{4}$ = 1011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000	0001
$\underline{S}_{3}$ = 1001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000	0000
$\underline{S}_{14}$ = 1000	0001	0011	0010	0110	0111	0101	0100	1100	1101	1111	1110	1010	1011	1001	1000

Figure 3.3 Decoder matrix for 16-DPSK signals.

C. RECEIVER LOCAL OSCILLATOR PHASE ERRORS

The introductory chapter summarizes the studies done on M-PSK DBDR structures, assuming that the receiver local oscillator has no phase error. In this research, a fixed receiver phase error has been assumed in the performance analyses conducted, and its effect on receiver BER is presented in Chapter IV and Chapter V.

IV. PERFORMANCE ANALYSES OF 8-PSK AND 8-DPSK DBDR'S WITH LOCAL OSCILLATOR PHASE ERROR

In previous chapters, DBDR's are introduced and their operational properties are explained. In this chapter, performance analyses for 8-PSK and 8-DPSK will be carried out in detail.

A. 8-PSK DBDR PERFORMANCE ANALYSIS IN THE PRESENCE OF RECEIVER LOCAL OSCILLATOR PHASE ERROR

The possible transmitted signals which are represented as vectors in Figure 1.2.a , can be mathematically expressed as

$$s_i(t) = \sqrt{2E_s/T_s}\,\sin[\,2\pi f_0 t + \theta_i(t)\,]\,, \qquad i=1,2,3...,8\,, \qquad 0\leq t \leq T_s \tag{4.1}$$

where

$$E_s = \int_0^{T_s} s_i^2(t)\,dt\,, \qquad i=1,2,..,8 \tag{4.2}$$

T_s = symbol duration

f_0 = carrier frequency (Hz.)

$\theta_i(t)$ indicates the phase of the transmitted PSK signal, which for the 8-PSK case becomes

$$\theta_i(t) = \begin{cases} \alpha + (i-1)\,\pi/4 & i=1,3,5,7 \\ i\pi/4 - \alpha & i=2,4,6,8 \end{cases} \qquad 0\leq t\leq T_s \tag{4.3}$$

The angle α is variable as shown in Figure 1.2.a; however, the constraint $0\leq\alpha\leq\pi/4$ must be observed. Equation 4.1 can be written by using a suitable orthonormal set of functions as

$$s_i(t) = \varphi_1(t)\sqrt{E_s}\cos\theta_i(t) + \varphi_2(t)\sqrt{E_s}\sin\theta_i(t) \tag{4.4}$$

$$i=1,2,\ldots,8 \; ; \; 0 \leq t \leq T_s$$

where $\varphi_1(t)$ and $\varphi_2(t)$ are the orthonormal function set which by applying the Gram-Schmitt orthonormalization procedure
[Ref. 3: p.198]
can be shown to be

$$\varphi_1(t) = \sqrt{2/T}\,\sin 2\pi f_0 t$$
$$\varphi_2(t) = \sqrt{2/T}\,\cos 2\pi f_0 t\,, \qquad 0 \leq t \leq T_s \tag{4.5}$$

The received signal depends upon the channel model and the transmitted signal as specified by Equation 4.1. In this research, an Additive White Gaussian Noise (AWGN) interference model has been assumed so that the received signal can be written as

$$r(t) = s_i(t) + n(t) \tag{4.6}$$

where n(t) is a sample function of a white Gaussian noise process having a constant (one sided) power spectral density function equal to N_0 watts/Hz. The receiver processes r(t) over an interval of duration T_s seconds and a decision is made at the end of the interval. This procedure continues in a repetitive fashion as time progresses.

Assuming all signals are equally likely to be transmitted, the probability of a bit being correct for M-PSK signal transmission is written as

$$\Pr\{\text{ bit correct }\} = 1/M \sum_{i=1}^{M} \Pr\{\text{ bit correct } / \; s_i(t) \text{ transmitted }\} \tag{4.7}$$

The calculations for the conditional probabilities ($\Pr\{$ bit correct / $s_i(t)$ transmitted $\}$, i=1,2,....M) have been determined by Myers and Bukofzer [Ref. 7: pp. 6-12], for the 8-PSK case with no receiver local oscillator phase error. In the sequel, the same analysis methods are utilized to obtain the receiver BER in the presence of a local oscillator phase error. A local oscillator phase error is modeled by the angle ε in the unmodulated receiver carriers $\varphi_1(t)$ and $\varphi_2(t)$ of Figure 1.6. These are denoted $\varphi_1'(t)$ and $\varphi_2'(t)$, namely

$$\varphi_1'(t) = \sqrt{2/T}\,\sin(\,2\pi f_0 t + \varepsilon\,)$$
$$\varphi_2'(t) = \sqrt{2/T}\,\cos(\,2\pi f_0 t + \varepsilon\,), \qquad 0 \le t \le T_s \tag{4.8}$$

Assuming signal $s_i(t)$ was transmitted, this produces outputs $r_{k,i}$ from the receiver's correlators, where

$$r_{k,i} = \int_0^{T_s} [\, s_i(t) + n(t)\,]\, \varphi_k'(t)\, dt \quad , \quad i = 1,2,...8 \quad , \quad k = 1,2 \tag{4.9}$$

For example when $s_1(t)$ is transmitted, from Equation 4.4, $\theta_i(t)$ becomes just α and

$$r_{1,1} = \sqrt{E_s}\,\cos(\alpha - \varepsilon) + n_1$$
$$r_{2,1} = \sqrt{E_s}\,\sin(\alpha - \varepsilon) + n_2 \tag{4.10}$$

where

$$n_k = \int_0^{T_s} n(t)\, \varphi_k'(t)\, dt\,, \qquad k = 1,2 \tag{4.11}$$

Observing that

$$E\,[n_1\, n_2] = E\,[\, \int_0^{T_s} n(t)\, \varphi_1'(t)\, dt \times \int_0^{T_s} n(t)\, \varphi_2'(t)\, dt\,] = 0 \tag{4.12}$$

This means that n_1 and n_2 are uncorrelated random variables. Since n_1 and n_2 are Gaussian random variables (having identical variances) [Ref. 7: p.7], they are also statistically independent.

The goal of the receiver of Figure 1.6 is to correctly recover the LB, the MB, and the RB, transmitted via the signal $s_i(t)$. By introducing the more compact notation

$$\Pr\{\text{ bit correct } / \; s_i(t)\,\} = \Pr\{\text{ bit correct } / \; s_i(t) \text{ transmitted }\} \quad , \; i = 1,2..,8 \tag{4.13}$$

the probabilities of such correct bit recovery are given in general by

$$\Pr\{\text{ LB correct } / s_i(t)\} = \begin{cases} \Pr\{ r_{1,i} > 0 / s_i(t) \} & i = 1,2,8,7 \\ \Pr\{ r_{1,i} < 0 / s_i(t) \} & i = 3,4,5,6 \end{cases} \tag{4.14}$$

$$\Pr\{\text{ MB correct } / s_i(t)\} = \begin{cases} \Pr\{ r_{1,i}^2 - r_{2,i}^2 > 0 / s_i(t) \} & i = 1,4,5,8 \\ \Pr\{ r_{1,i}^2 - r_{2,i}^2 < 0 / s_i(t) \} & i = 2,3,6,7 \end{cases} \tag{4.15}$$

$$\Pr\{\text{ RB correct } / s_i(t)\} = \begin{cases} \Pr\{ r_{2,i} > 0 / s_i(t) \} & i = 1,2,3,4 \\ \Pr\{ r_{2,i} < 0 / s_i(t) \} & i = 5,6,7,8 \end{cases} \tag{4.16}$$

From Equations 4.8-4.16, we can write

$$\begin{aligned} \Pr\{\text{ LB correct } / s_1(t)\} &= \Pr\{ r_{1,1} > 0 / s_1(t) \} \\ &= \Pr\{ n_1 > -\sqrt{E_s} \cos(\alpha-\varepsilon) / s_1(t) \} \\ &= 1 - Q[\sqrt{2 E_s/N_0} \cos(\alpha-\varepsilon)] \end{aligned} \tag{4.17}$$

where

$$Q(x) = \int_x^{\infty} [1/\sqrt{2\pi}] \exp(-u^2/2)\, du \tag{4.18}$$

Similarly

$$\begin{aligned} \Pr\{\text{ RB correct } / s_1(t)\} &= \Pr\{ r_{2,1} > 0 / s_1(t) \} \\ &= \Pr\{ n_2 > -\sqrt{E_s} \sin(\alpha-\varepsilon) / s_1(t) \} \\ &= 1 - Q[\sqrt{2 E_s/N_0} \sin(\alpha-\varepsilon)] \end{aligned} \tag{4.19}$$

In order to evaluate the conditional probability of the MB being correct, the following steps are required. From Equation 4.15

$$\begin{aligned} \Pr\{\text{ MB correct } / s_i(t)\} &= \Pr\{ r_{1,i}^2 - r_{2,i}^2 > 0 / s_i(t) \} \\ &= \Pr\{(r_{1,i} - r_{2,i}) \cdot (r_{1,i} + r_{2,i}) > 0 / s_i(t) \} \end{aligned} \tag{4.20}$$

By defining variables $y_{1,i}$ and $y_{2,i}$ as

$$\begin{aligned} y_{1,i} &= r_{1,i} + r_{2,i} \\ y_{2.i} &= r_{1,i} - r_{2,i} \end{aligned} \qquad (4.21)$$

then from Equation 4.9, when $s_1(t)$ is transmitted,

$$y_{1,1} = \sqrt{E_s}\,[\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)] + n_1 + n_2 \qquad (4.22)$$

$$y_{2,1} = \sqrt{E_s}\,[\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)] + n_1 - n_2 \qquad (4.23)$$

Clearly $y_{1,2}$ and $y_{2,1}$ are Gaussian random variables with statistical means given by

$$E\{y_{1,1}\} = \sqrt{E_s}\,[\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)] \qquad (4.24)$$

$$E\{y_{2,1}\} = \sqrt{E_s}\,[\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)] \qquad (4.25)$$

and variances given by

$$\mathrm{Var}\{y_{1,1}\} = \mathrm{Var}\{y_{2,1}\} = N_0 \qquad (4.26)$$

By observing that

$$\begin{aligned} E\{(y_{1,1} - E[y_{1,1}])\,(y_{2,1} - E[y_{2,1}])\} &= E\{(n_1 + n_2)(n_1 - n_2)\} \\ &= E\{(n_1)^2 - (n_2)^2\} = 0 \end{aligned} \qquad (4.27)$$

where n_1 and n_2 are defined in Equation 4.11. Since $y_{1,1}$ and $y_{2,1}$ are uncorrelated Gaussian random variables, they are also statistically independent. Therefore, Equation 4.20 becomes

$$\begin{aligned} \Pr\{\text{MB correct} / s_1(t)\} &= \Pr\{y_{1,1}\ y_{2,1} > 0 / s_1(t)\} \\ &= \Pr\{[y_{1,1} > 0 / s_1(t)] \text{ and } [y_{2,1} > 0 / s_1(t)]\} + \\ &+ \Pr\{[y_{1,1} < 0 / s_1(t)] \text{ and } [y_{2,1} < 0 / s_1(t)]\} + \\ &= \Pr\{y_{1,1} > 0 / s_1(t)\}\Pr\{y_{2,1} > 0 / s_1(t)\} + \\ &+ \Pr\{y_{1,1} < 0 / s_1(t)\}\Pr\{y_{2,1} < 0 / s_1(t)\} \end{aligned} \qquad (4.28)$$

From Equations 4.22 - 4.26, we have

$$\Pr\{\, y_{1,1} > 0 \,/\, s_1(t) \,\} = Q\,[\, E\{y_{1,1}\} \,/\, \sqrt{N_0}\,] \qquad (4.29)$$

$$\Pr\{\, y_{2,1} > 0 \,/\, s_1(t) \,\} = Q\,[\, E\{y_{2,1}\} \,/\, \sqrt{N_0}\,] \qquad (4.30)$$

so that Equation 4.28 becomes

$$\begin{aligned} \Pr\{\text{ MB correct } / s_1(t) \} = {} & 1 - Q\{ \sqrt{E_s/N_0}\,[\,\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon]\,\} - \\ & - Q\{ \sqrt{E_s/N_0}\,[\,\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon]\,\} \\ & - 2\,Q\{ \sqrt{E_s/N_0}\,[\,\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon]\,\} \,. \\ & .\, Q\{ \sqrt{E_s/N_0}\,[\,\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon]\,\} \qquad (4.31) \end{aligned}$$

Assuming now transmission of $s_2(t)$, a similar analysis procedure is performed but now Equations 4.14 - 4.16 become

$$\theta_2(t) = \pi/2 - \alpha \qquad (4.32)$$

$$\begin{aligned} r_{1,2} &= \sqrt{E_s}\,\sin(\alpha+\varepsilon) + n_1 \\ r_{2,2} &= \sqrt{E_s}\,\cos(\alpha+\varepsilon) + n_2 \qquad (4.33) \end{aligned}$$

where n_1 and n_2 are defined by Equation 4.11. From Equations 4.14 and 4.33

$$\begin{aligned} \Pr\{\text{ LB correct } / s_2(t) \} &= \Pr\{\, r_{1,2} > 0 \,/\, s_2(t) \,\} \\ &= \Pr\{\, n_2 > -\sqrt{E_s}\,\sin(\alpha+\varepsilon) \,/\, s_2(t) \,\} \\ &= 1 - Q\,[\, \sqrt{2\,E_s/N_0}\;\sin(\alpha+\varepsilon)\,] \qquad (4.34) \end{aligned}$$

and

$$\begin{aligned} \Pr\{\text{ RB correct } / s_2(t) \} &= \Pr\{\, r_{2,2} > 0 \,/\, s_2(t) \,\} \\ &= \Pr\{\, n_2 > -\sqrt{E_s}\,\cos(\alpha+\varepsilon) \,/\, s_2(t) \,\} \\ &= 1 - Q\,[\, \sqrt{2\,E_s/N_0}\;\cos(\alpha+\varepsilon)\,] \qquad (4.35) \end{aligned}$$

For calculations involving the MB, one additional random variable needs to be defined, namely

$$y_{3,i} = r_{2,i} - r_{1,i} \qquad i = 1,2,\ldots.8 \qquad (4.36)$$

Assuming transmission of $s_2(t)$, Equation 4.36 becomes

$$\begin{aligned} y_{3,2} &= r_{2,2} - r_{1,2} \\ &= \sqrt{E_s} \cos(\alpha+\varepsilon) + n_2 - \sqrt{E_s} \sin(\alpha+\varepsilon) - n_1 \end{aligned} \tag{4.37}$$

where

$$E\{y_{3,2}\} = \sqrt{E_s} \cos(\alpha+\varepsilon) - \sqrt{E_s} \sin(\alpha+\varepsilon) = E\{y_{2,1}\} \tag{4.38}$$

Since

$$\begin{aligned} \Pr\{ \text{MB correct} / s_2(t) \} &= \Pr\{ (r_{1,1})^2 - (r_{2,2})^2 < 0 / s_2(t) \} \\ &= \Pr\{ (r_{1,1} - r_{2,2})\ (r_{1,1} + r_{2,2}) < 0 \ / s_2(t) \} \end{aligned} \tag{4.39}$$

by using Equations 4.21 and 4.36

$$\Pr\{ \text{MB correct} / s_2(t) \} = \Pr\{ y_{1,2}\, y_{3,2} < 0 / s_2(t) \} \tag{4.40}$$

Furthermore

$$\begin{aligned} E\{ (y_{1,2} - E\{ y_{1,2}\})\ (y_{3,2} - E\{ y_{3,2}\}) \} &= E\{ (n_1 + n_2)\ (n_1 - n_2) \} \\ &= E\{ (n_1)^2 - (n_2)^2 \} = 0 \end{aligned} \tag{4.41}$$

so that $y_{1,2}$ and $y_{3,2}$ are statistically independent Gaussian random variables, and therefore Equation 4.40 becomes

$$\begin{aligned} \Pr\{ \text{MB correct} / s_2(t) \} &= \Pr\{ y_{3,2}\ y_{1,2} < 0 / s_2(t) \} \\ &= \Pr\{ [\, y_{1,2} > 0 / s_1(t) \,] \text{ and } [\, y_{3,2} > 0 / s_2(t) \,] \} + \\ &+ \Pr\{ [\, y_{1,2} < 0 / s_2(t) \,] \text{ and } [\, y_{3,2} < 0 / s_2(t) \,] \} + \\ &= \Pr\{ y_{1,2} > 0 / s_2(t) \} \Pr\{ y_{3,2} > 0 / s_2(t) \} + \\ &+ \Pr\{ y_{1,2} < 0 / s_2(t) \} \Pr\{ y_{3,2} < 0 / s_2(t) \} \end{aligned} \tag{4.42}$$

By using a method similar to that used in the derivation of Equations 4.29 and 4.30, for $y_{1,2}$ and $y_{3,2}$, Equation 4.42 becomes

$$\begin{aligned}
\Pr\{ \text{ MB correct } / \ s_2(t) \} = \ & 1 - Q\{ \sqrt{E_s/N_0}\,[\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon] \} - \\
& - Q\{ \sqrt{E_s/N_0}\,[\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon] \} \\
& - 2\,Q\{ \sqrt{E_s/N_0}\,[\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon] \} \, . \\
& . \, Q\{ \sqrt{E_s/N_0}\,[\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon] \} \qquad (4.43)
\end{aligned}$$

Analyses can be carried out for probability of correct reception assuming every possible signal transmission. This is however not necessary because of inherent symmetries in the problem . In conditions where symmetries are not present, previous methodologies can be used to obtain results as follows. Assuming transmission of $s_3(t)$, Equations 4.3 and 4.9 become

$$\theta_3(t) = \pi/2 + \alpha \qquad (4.44)$$

$$\begin{aligned}
r_{1,3} &= \sqrt{E_s}\cos(\alpha-\varepsilon+\pi/2) + n_1 \qquad (4.45) \\
&= -\sqrt{E_s}\sin(\alpha-\varepsilon) + n_1
\end{aligned}$$

$$\begin{aligned}
r_{2,3} &= \sqrt{E_s}\sin(\alpha-\varepsilon+\pi/2) + n_2 \qquad (4.46) \\
&= -\sqrt{E_s}\cos(\alpha-\varepsilon) + n_2
\end{aligned}$$

so that from Equation 4.14

$$\begin{aligned}
\Pr\{ \text{ LB correct } / \ s_3(t) \} &= \Pr\{ r_{1,3} < 0 \ / \ s_3(t) \} \\
&= \Pr\{ n_1 > -\sqrt{E_s}\sin(\alpha-\varepsilon) \ / \ s_3(t) \} \\
&= 1 - Q[\sqrt{2\,E_s/N_0}\,\sin(\alpha-\varepsilon)\,] \qquad (4.47)
\end{aligned}$$

and from Equation 4.16

$$\begin{aligned}
\Pr\{ \text{ RB correct } / \ s_3(t) \} &= \Pr\{ r_{2,3} < 0 \ / \ s_3(t) \} \\
&= \Pr\{ n_2 > -\sqrt{E_s}\cos(\alpha-\varepsilon) \ / \ s_3(t) \} \\
&= 1 - Q[\sqrt{2\,E_s/N_0}\,\cos(\alpha+\varepsilon)\,] \qquad (4.48)
\end{aligned}$$

Assuming transmission of $s_4(t)$, Equations 4.3 and 4.4 become

$$\theta_4(t) = \pi - \alpha \qquad (4.49)$$

$$\begin{aligned}
r_{1,4} &= \sqrt{E_s}\cos(\pi-\alpha-\varepsilon) + n_1 \qquad (4.50) \\
&= -\sqrt{E_s}\cos(\alpha+\varepsilon) + n_1
\end{aligned}$$

$$\begin{aligned} r_{2,4} &= \sqrt{E_s}\,\sin(\pi-\alpha-\varepsilon) + n_2 \\ &= -\sqrt{E_s}\,\sin(\alpha+\varepsilon) + n_2 \end{aligned} \tag{4.51}$$

so that from Equation 4.14

$$\begin{aligned} \Pr\{\text{ LB correct } / \ s_4(t)\ \} &= \Pr\{\ r_{1,4} < 0 \ / \ s_4(t)\ \} \\ &= \Pr\{\ n_1 > -\sqrt{E_s}\,\sin(\alpha+\varepsilon) \ / \ s_4(t)\ \} \\ &= 1 - Q[\ \sqrt{2\,E_s/N_0}\,\cos(\alpha+\varepsilon)\ ] \end{aligned} \tag{4.52}$$

and from Equation 4.16

$$\begin{aligned} \Pr\{\text{ RB correct } / \ s_4(t)\ \} &= \Pr\{\ r_{2,4} > 0 \ / \ s_4(t)\ \} \\ &= \Pr\{\ n_2 > -\sqrt{E_s}\,\sin(\alpha+\varepsilon) \ / \ s_4(t)\ \} \\ &= 1 - Q[\ \sqrt{2\,E_s/N_0}\,\sin(\alpha+\varepsilon)\ ] \end{aligned} \tag{4.53}$$

Also from Equation 4.15

$$\begin{aligned} \Pr\{\text{ MB correct } / \ s_4(t)\ \} &= \Pr\{\ (r_{1,4})^2 - (r_{2,4})^2 > 0 \ / \ s_4(t)\ \} \\ &= \Pr\{\ (r_{1,4} - r_{2,4})\ (r_{1,4} + r_{2,4}) > 0 \ / \ s_4(t)\ \} \end{aligned} \tag{4.54}$$

Using Equations 4.21 and 4.36 yields

$$y_{1,4} = r_{1,4} + r_{2,4} \tag{4.55}$$

$$y_{2,4} = r_{1,4} - r_{2,4} \tag{4.56}$$

and using a method similar to that used in the derivation of Equation 4.27 results in

$$\begin{aligned} \Pr\{\text{ MB correct } / \ s_4(t)\ \} &= \Pr\{\ y_{1,4}\ y_{2,4} > 0 \ / \ s_4(t)\ \} \\ &= \Pr\{\ [\ y_{1,4} > 0 \ / \ s_4(t)\ ] \text{ and } [\ y_{2,4} > 0 \ / \ s_4(t)\ ]\ \} + \\ &+ \Pr\{\ [\ y_{1,4} < 0 \ / \ s_4(t)\ ] \text{ and } [\ y_{2,4} < 0 \ / \ s_4(t)\ ]\ \} + \\ &= \Pr\{\ y_{1,4} > 0 \ / \ s_4(t)\ \}\ \Pr\{\ y_{2,4} > 0 \ / \ s_4(t)\ \} + \\ &+ \Pr\{\ y_{1,4} < 0 \ / \ s_4(t)\ \}\ \Pr\{\ y_{2,4} < 0 \ / \ s_4(t)\ \} \end{aligned} \tag{4.57}$$

which becomes

$$
\begin{aligned}
\Pr\{\text{ MB correct } / \; s_4(t)\} = \; & 1 - Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)]\} - \\
& - Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)]\} + \\
& + 2\,Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)]\} \,. \\
& .\, Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)]\}
\end{aligned} \tag{4.58}
$$

For transmission of $s_5(t)$, the pertinent equations are

$$\theta_5(t) = \pi + \alpha \tag{4.59}$$

$$
\begin{aligned}
r_{1,5} &= \sqrt{E_s}\cos(\pi+\alpha-\varepsilon) + n_1 \\
&= -\sqrt{E_s}\cos(\alpha-\varepsilon) + n_1
\end{aligned} \tag{4.60}
$$

$$
\begin{aligned}
r_{2,5} &= \sqrt{E_s}\sin(\pi+\alpha-\varepsilon) + n_2 \\
&= -\sqrt{E_s}\sin(\alpha-\varepsilon) + n_2
\end{aligned} \tag{4.61}
$$

so that

$$
\begin{aligned}
\Pr\{\text{ MB correct } / \; s_5(t)\} &= \Pr\{(r_{1,5})^2 - (r_{2,5})^2 > 0 \;/\; s_5(t)\} \\
&= \Pr\{(r_{1,5} - r_{2,5})\;(r_{1,5} + r_{2,5}) > 0 \;/\; s_5(t)\}
\end{aligned} \tag{4.62}
$$

which becomes

$$
\begin{aligned}
\Pr\{\text{ MB correct } / \; s_5(t)\} &= \Pr\{y_{1,5}\; y_{2,5} > 0 \;/\; s_5(t)\} \\
&= \Pr\{[\,y_{1,5} > 0 \;/\; s_5(t)\,] \text{ and } [\,y_{2,5} > 0 \;/\; s_5(t)\,]\} + \\
&+ \Pr\{[\,y_{1,5} < 0 \;/\; s_5(t)\,] \text{ and } [\,y_{2,5} < 0 \;/\; s_5(t)\,]\} + \\
&= \Pr\{y_{1,5} > 0 \;/\; s_5(t)\}\,\Pr\{y_{2,5} > 0 \;/\; s_5(t)\} + \\
&+ \Pr\{y_{1,5} < 0 \;/\; s_5(t)\}\,\Pr\{y_{2,5} < 0 \;/\; s_5(t)\}
\end{aligned} \tag{4.57}
$$

by observing that $y_{1,5}$ and $y_{2,5}$ are statistically independent random variables making it possible to apply the similar procedure shown above. The result becomes

$$
\begin{aligned}
\Pr\{\text{ MB correct } / \; s_5(t)\} = \; & 1 - Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)]\} - \\
& - Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)]\} + \\
& + 2\,Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)]\} \,. \\
& .\, Q\{\sqrt{2\,E_s/N_0}\,[\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)]\}
\end{aligned} \tag{4.64}
$$

As indicated before, because of the inherent symmetry in the problem, the following equations hold for conditional LB correct probabilities (see Figure 1.2.b)

$$\Pr\{ \text{LB correct} / s_1(t) \} = \Pr\{ \text{LB correct} / s_5(t) \} \tag{4.65}$$
$$\Pr\{ \text{LB correct} / s_2(t) \} = \Pr\{ \text{LB correct} / s_6(t) \} \tag{4.66}$$
$$\Pr\{ \text{LB correct} / s_3(t) \} = \Pr\{ \text{LB correct} / s_7(t) \} \tag{4.67}$$
$$\Pr\{ \text{LB correct} / s_4(t) \} = \Pr\{ \text{LB correct} / s_8(t) \} \tag{4.68}$$

and similarly for the RB

$$\Pr\{ \text{RB correct} / s_1(t) \} = \Pr\{ \text{RB correct} / s_5(t) \} \tag{4.69}$$
$$\Pr\{ \text{RB correct} / s_2(t) \} = \Pr\{ \text{RB correct} / s_6(t) \} \tag{4.70}$$
$$\Pr\{ \text{RB correct} / s_3(t) \} = \Pr\{ \text{RB correct} / s_7(t) \} \tag{4.71}$$
$$\Pr\{ \text{RB correct} / s_4(t) \} = \Pr\{ \text{RB correct} / s_8(t) \} \tag{4.72}$$

and finally for the MB

$$\Pr\{ \text{MB correct} / s_1(t) \} = \Pr\{ \text{MB correct} / s_3(t) \} \tag{4.75}$$
$$\Pr\{ \text{MB correct} / s_2(t) \} = \Pr\{ \text{MB correct} / s_8(t) \} \tag{4.76}$$
$$\Pr\{ \text{MB correct} / s_4(t) \} = \Pr\{ \text{MB correct} / s_6(t) \} \tag{4.77}$$
$$\Pr\{ \text{MB correct} / s_5(t) \} = \Pr\{ \text{MB correct} / s_7(t) \} \tag{4.78}$$

By using now Equation 4.7 for the unconditional probability of the LB, MB and RB being correct, the above results yield

$$\begin{aligned} \Pr\{ \text{RB correct} \} &= \Pr\{ \text{LB correct} \} \\ &= 1 - 1/4 \, [\, Q[\sqrt{2 E_s/N_0} \cos(\alpha - \varepsilon)] + \\ &+ Q[\sqrt{2 E_s/N_0} \sin(\alpha + \varepsilon)] + \\ &+ Q[\sqrt{2 E_s/N_0} \cos(\alpha + \varepsilon)] \\ &+ Q[\sqrt{2 E_s/N_0} \sin(\alpha - \varepsilon)] \,] \end{aligned} \tag{4.79}$$

and

$$\begin{aligned} \Pr\{ \text{MB correct} \} &= 1 - 1/2 \, [\; Q \{ \sqrt{2 E_s/N_0} \, [\cos(\alpha - \varepsilon) + \sin(\alpha - \varepsilon)] \} + \\ &+ Q \{ \sqrt{2 E_s/N_0} \, [\cos(\alpha + \varepsilon) + \sin(\alpha + \varepsilon)] \} + \end{aligned}$$

$$
\begin{aligned}
&+ Q\left\{ \sqrt{2\,E_s/N_0}\,[\,\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)\,] \right\} + \\
&+ Q\left\{ \sqrt{2\,E_s/N_0}\,[\,\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)\,] \right\} \,]+ \\
&+ Q\left\{ \sqrt{2\,E_s/N_0}\,[\,\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)\,] \right\} \cdot \\
&\cdot Q\left\{ \sqrt{2\,E_s/N_0}\,[\,\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)\,] \right\} + \\
&+ Q\left\{ \sqrt{2\,E_s/N_0}\,[\,\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)\,] \right\} \cdot \\
&\cdot Q\left\{ \sqrt{2\,E_s/N_0}\,[\,\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)\,] \right\} \,] \qquad (4.80)
\end{aligned}
$$

For 8-PSK,the Signal to Noise Ratio (SNR) has been defined by Equation 1.6, so that using Equations 4.79 - 4.80, the BER denoted PE can be expressed as a function of SNR, α and ε as follows

$$
\begin{aligned}
PE &= 1/3\,[\,\Pr\{\text{ LB correct }\} + \Pr\{\text{ MB correct }\} + \Pr\{\text{ RB correct }\}\,] \\
&= 1 - 1/6\,\{\, Q[\,2\sqrt{SNR}\cos(\alpha-\varepsilon)\,] + Q[\,2\sqrt{SNR}\sin(\alpha+\varepsilon)\,] + \\
&+ Q[\,2\sqrt{SNR}\cos(\alpha+\varepsilon)\,] + Q[\,2\sqrt{SNR}\sin(\alpha-\varepsilon)\,] + \\
&+ Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)\,]\,\} + \\
&+ Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)\,]\,\} + \\
&+ Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)\,]\,\} + \\
&+ Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)\,]\,\}\,\} + \\
&+ Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)\,]\,\} \cdot \\
&\cdot Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)\,]\,\} + \\
&+ Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)\,]\,\} \cdot \\
&\cdot Q\{\,2\sqrt{SNR}\,[\,\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)\,]\,\} \qquad (4.81)
\end{aligned}
$$

From Equation 4.81, the PE can be computed for various values of SNR, α, and ε, with the results used to optimize the DBDR performance. For $\varepsilon = 0^\circ$ (i. e. , no receiver local oscillator phase error), values of PE vs SNR have been plotted (see Figure 4.1) showing complete agreement with the results obtained by Myers and Bukofzer [Ref. 7: p. 11] Figure 4.1 includes also the PE for various values of ε, as a function of SNR.

B. 8-DPSK DBDR PERFORMANCE ANALYSIS IN THE PRESENCE OF RECEIVER LOCAL OSCILLATOR PHASE ERROR

The advantages and disadvantages of differential encoding and decoding have been previously explained in the introductory chapter. In this section, performance of the 8-DPSK DBDR is presented. As explained earlier the 8-DPSK decoder utilizes the

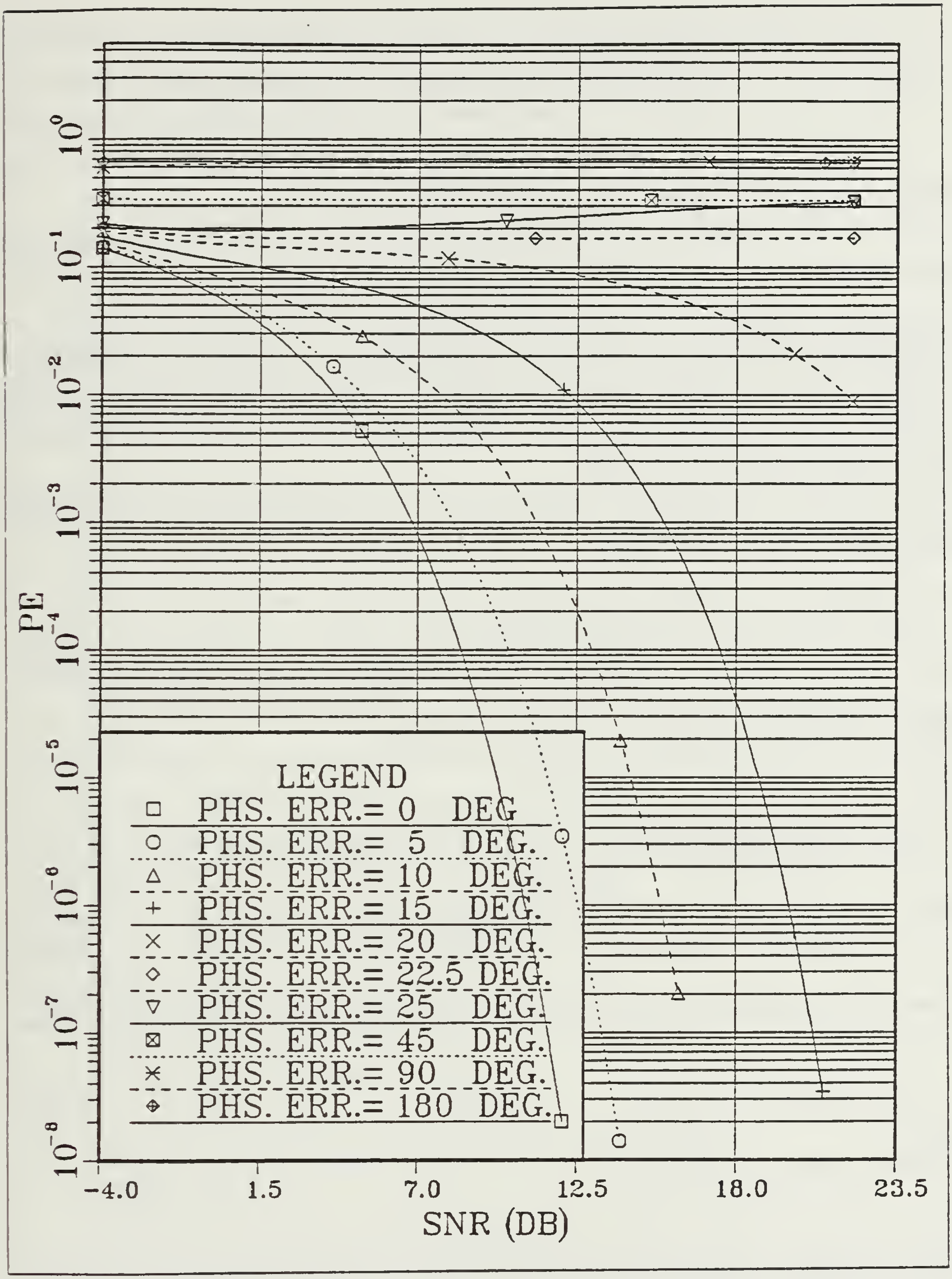

Figure 4.1 PE Versus SNR of 8-PSK DBDR for Various Values of ε (α = 22.5°).

LB, MB and RB, which are recovered via direct bit detection methods, and performs an inverse mapping from which the original data stream is delivered to the intended user

Before proceeding with the performance calculations, the following notation will be introduced. The probabilities of a bit being correct or being in error will be denoted

$$Ck = \Pr\{ \text{ k correct } \} \ , \qquad Ek = \Pr\{ \text{ k error } \} \tag{4.82}$$

respectively, where k= 1 for LB, k= 2 for MB, k= 3 for RB. By introducing the notation given by Equation 4.82, the analysis for the performance of the 8-DPSK DBDR can be expressed in more compact form. For example E2 means the probability that the MB is in error.

In order to obtain the probability of a bit being correct (with differential decoding) we must first evaluate

$$\begin{aligned} \Pr\{ \text{ RB correct } \} &= 1/2 \Pr\{ \text{ RB correct } / \text{ RB}= 1 \} + \\ &+ 1/2 \Pr\{ \text{ RB correct } / \text{ RB}= 0 \} \end{aligned} \tag{4.83}$$

where by further conditioning, we equivalently have

$$\begin{aligned} \Pr\{ \text{ RB correct/RB}=\text{m} \} = \sum_{i=0}^{1} \sum_{j=0}^{1} & \Pr\{ \text{ RB correct } / \text{ RB}=\text{m,MB}=\text{i,LB}=\text{j} \}. \\ & . \Pr\{ \text{ MB}=\text{i, LB}=\text{j} \} \qquad \text{m}=0,1 \end{aligned} \tag{4.84}$$

Due to the assumption of equally likely probabilities of having 0's and 1's in the data,

$$\Pr\{ \text{ MB}=\text{i, LB}=\text{j} \} = 1/4 \qquad \text{i,j}= 0,1 \tag{4.85}$$

so that Equation 4.83 becomes

$$\Pr\{ \text{ RB correct } \} = 1/8 \sum_{i=0}^{1} \sum_{j=0}^{1} \Pr\{ \text{ RB correct } / \text{ RB}=0\text{,MB}=\text{i,LB}=\text{j} \} +$$

$$+ 1/8 \sum_{i=0}^{1} \sum_{j=0}^{1} \Pr\{ \text{RB correct} / \text{RB}=1, \text{MB}=i, \text{LB}=j \} \qquad (4.86)$$

The DBDRs recover from the received signal, each bit in the data state (for PSK modulation) or each bit in the symbol state (for DPSK modulation) independently. This property of the DBDR is an advantage for BER analyses. Because certain consecutive bit errors in the past and presently recovered symbol states do not necessarily result in errors in the recovered data states, these bit errors in successive symbol states must be considered. That is, even though symbol errors may occur, the delivered data may stillbe correct. Therefore, these type errors have to be accounted for as resulting in a correct reception and considered in the computation of the probability of a bit being correct. Figure 4.2 shows one example of the situation being discussed. Here the data state (0,0,0) is assumed to occur and the symbol state (0,0,0) is assumed to have been previously transmitted. This means that according to the encoder matrix shown in Chapter II, the symbol state (0,0,0) must now be generated for transmission. Figure 4.2 further shows the receptions of the symbol states which even though, most contain errors, produce recovered data states in which the RB is correctly delivered. Observe that there are 32 such combinations which yield a correct 0 for the RB. Therefore notation has to be introduced in order to account for error patterns of the above type, namely Pr{ RB correct, (k,l,m) / RB=0,MB=i,LB=j }, where k,l,m= 0,1. The meaning of this expression is the probability of the RB correctly recovered given that the data state was (j,i,0) and the delivered data state is (k,l,m) (where m=0 in this case, otherwise the RB could not be correct due to the transmission of the RB=0). Therefore

$$\Pr\{ \text{RB correct} / \text{RB}=0, \text{MB}=0, \text{LB}=0 \} =$$

$$= \sum_{k=0}^{1} \sum_{l=0}^{1} \Pr\{ \text{RB correct}, (k,l,0) / \text{RB}=0, \text{MB}=0, \text{LB}=0 \} \qquad (4.87)$$

Since each element of the symbol state is independently recovered, we see from Figure 4.2 that, for example

```
################################################################################
       CORRECT RECEPTION COMBINATIONS IN PRESENCE OF ERROR
################################################################################

    INPUT       PREVIOUS      PRESENT        PREVIOUS      PRESENT        OUTPUT
             TRANSMISSION TRANSMISSION    RECEPTION     RECEPTION
--------------------------------------------------------------------------------
    0  0  0     0  0  0      0  0  0      0  0  0       0  0  0       0  0  0
                                          0  0  1       0  0  1       0  0  0
                                          0  1  0       0  1  0       0  0  0
                                          0  1  1       0  1  1       0  0  0
                                          1  0  0       1  0  0       0  0  0
                                          1  0  1       1  0  1       0  0  0
                                          1  1  0       1  1  0       0  0  0
                                          1  1  1       1  1  1       0  0  0
                                          0  0  0       1  1  1       0  1  0
                                          0  0  1       0  0  0       0  1  0
                                          0  1  0       1  1  0       0  1  0
                                          0  1  1       1  0  0       0  1  0
                                          1  0  0       1  0  1       0  1  0
                                          1  0  1       0  1  0       0  1  0
                                          1  1  0       0  0  1       0  1  0
                                          1  1  1       0  1  1       0  1  0
                                          0  0  0       0  1  0       1  0  0
                                          0  0  1       1  0  1       1  0  0
                                          0  1  0       0  1  1       1  0  0
                                          0  1  1       0  0  1       1  0  0
                                          1  0  0       0  0  0       1  0  0
                                          1  0  1       1  1  1       1  0  0
                                          1  1  0       1  0  0       1  0  0
                                          1  1  1       1  1  0       1  0  0
                                          0  0  0       1  0  1       1  1  0
                                          0  0  1       1  0  0       1  1  0
                                          0  1  0       1  1  1       1  1  0
                                          0  1  1       1  1  0       1  1  0
                                          1  0  0       0  0  1       1  1  0
                                          1  0  1       0  0  0       1  1  0
                                          1  1  0       0  1  1       1  1  0
                                          1  1  1       0  1  0       1  1  0
```

Figure 4.2 Example of Error Patterns for 8-DPSK System Where Data State, Previous Transmission, and Present Transmission are 000 with RB Correctly Recovered.

$$\Pr\{\text{ RB correct},(0,0,0) / RB=0,MB=0,LB=0 \} = C1^2\, C2^2\, C3^2 + C1^2\, C2^2\, E3^2 + C1^2\, E2^2\, C3^2 + C1^2\, E2^2\, E3^2 + E1^2\, C2^2\, C3^2 + E1^2\, C2^2\, E3^2 + E1^2\, E2^2\, C3^2 + E1^2\, E2^2\, E3^2 \tag{4.88}$$

From Equations 4.79 and 4.82 - 4.86, it is clear that $C1 = C3$ and $E1 = E3$. Therefore, simplification of Equation 4.88 is possible, thus yielding

$$\Pr\{\text{ RB correct},(0,0,0) / RB=0,MB=0,LB=0 \} = C1^4\, C2^2 + 2\, C1^2\, C2^2\, E1^2 + C1^4\, E2^2 + 2\, C1^2\, E1^2\, E2^2 + E1^4\, C2^2 + E1^4\, E2^2 \tag{4.89}$$

A similar expression must be obtained for other combinations of values of k and l as well as the condition that the RB is 1. This produces a total of 64 possibilities of received symbol states that result in a correct RB. A computer generated table was used ,in order to identify the equivalent error patterns for other cases involving MB = 1 and LB = 1. The appropriate listing are shown in Appendix C. Similar analyses must be carried out in order to determine the unconditional probabilities Pr{MB correct} as well as Pr{LB correct}. Such analyses demonstrate that due to the inherent symmetries

$$\Pr\{\text{ LB correct }\} = \Pr\{\text{ MB correct }\} = \Pr\{\text{ RB correct }\} \equiv PE \tag{4.90}$$

so that using Equation 4.86 along with similar forms obtained for $k = 1$ and $l = 1$ yield

$$\begin{aligned} PE = 1 - 1/2\, [\, & 2C1^4\, C2^2 + 9\, C1^2\, C2^2\, E1^2 + C1^4\, E2^2 + 3\, C1^4\, E2\, C2 + \\ & + 10\, C1^2\, E1^2\, E2^2 + 2\, E1^4\, C2^2 + E1^4\, E2^2 + 17\, C1^2\, C2\, E1^2\, E2 + \\ & + C1^3\, C2^2\, E1 + 3\, C1^3\, E1\, E2^2 + 2\, C1\, E1^3\, C2^2 + 2\, C1\, E1^3\, E2^2 + \\ & + 3\, C2\, E1^4\, E2 + 4\, C1\, C2\, E1^3\, E2 + 4\, C1^3\, C2\, E1\, E2\,] \end{aligned} \tag{4.91}$$

In Figures 4.4 and 4.5, PE (given by Equation 4.91) vs SNR have been plotted for various values of α and ε respectively.

C. NUMERICAL RESULTS AND DISCUSSION OF RESULTS FOR 8-PSK AND 8-DPSK DBDR'S WITH RECEIVER LOCAL OSCILLATOR PHASE ERROR.

Previous sections of this chapter have focused on the performance of 8-PSK and 8-DPSK DBDRs operating in the presence of a receiver local oscillator phase error. Without any phase error, we have shown the performance plots of 8-PSK and 8-DPSK

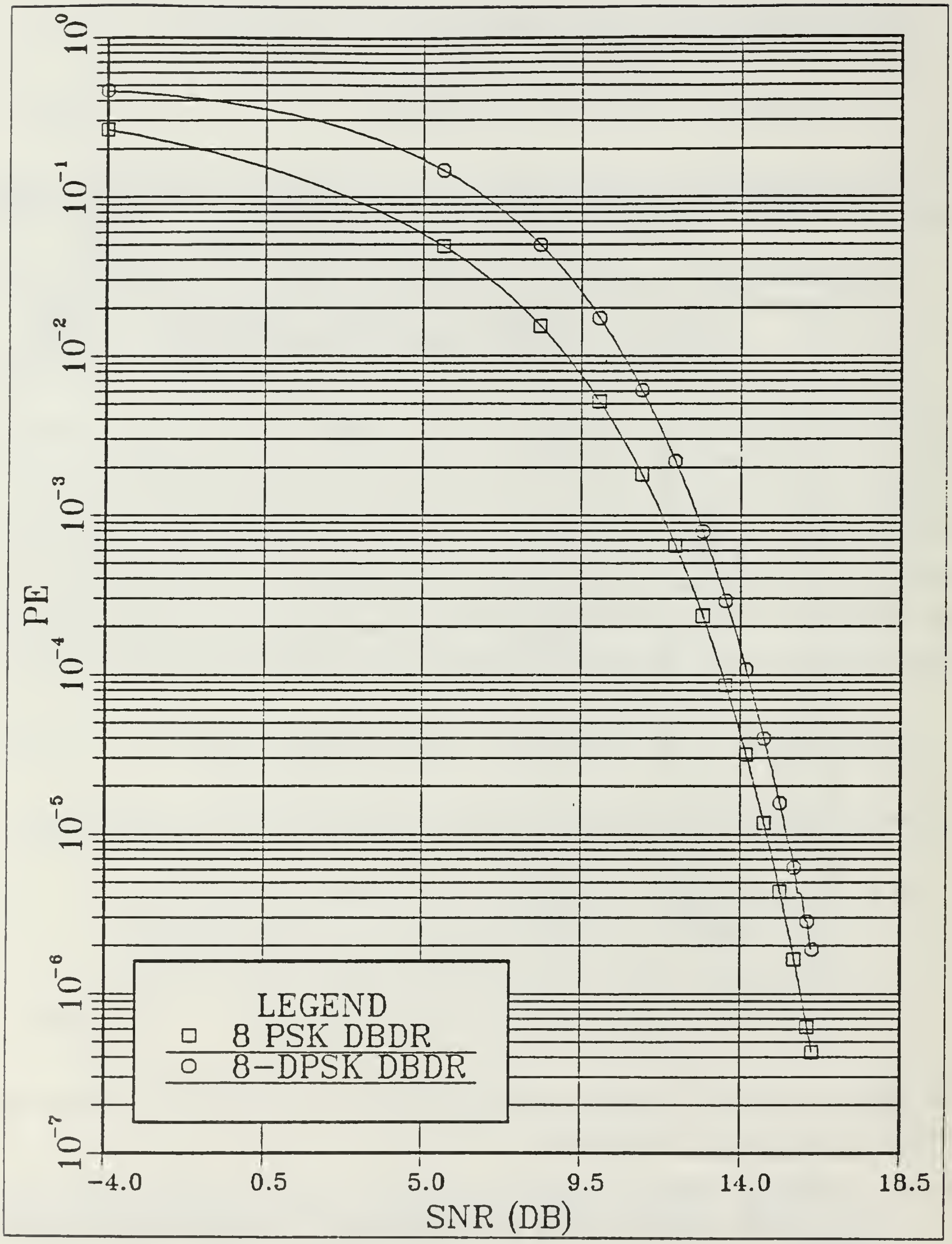

Figure 4.3 PE vs SNR Comparison Plot for 8-PSK and 8-DPSK DBDR ($\alpha = 22.5^{\circ}$, $\varepsilon = 0^{\circ}$).

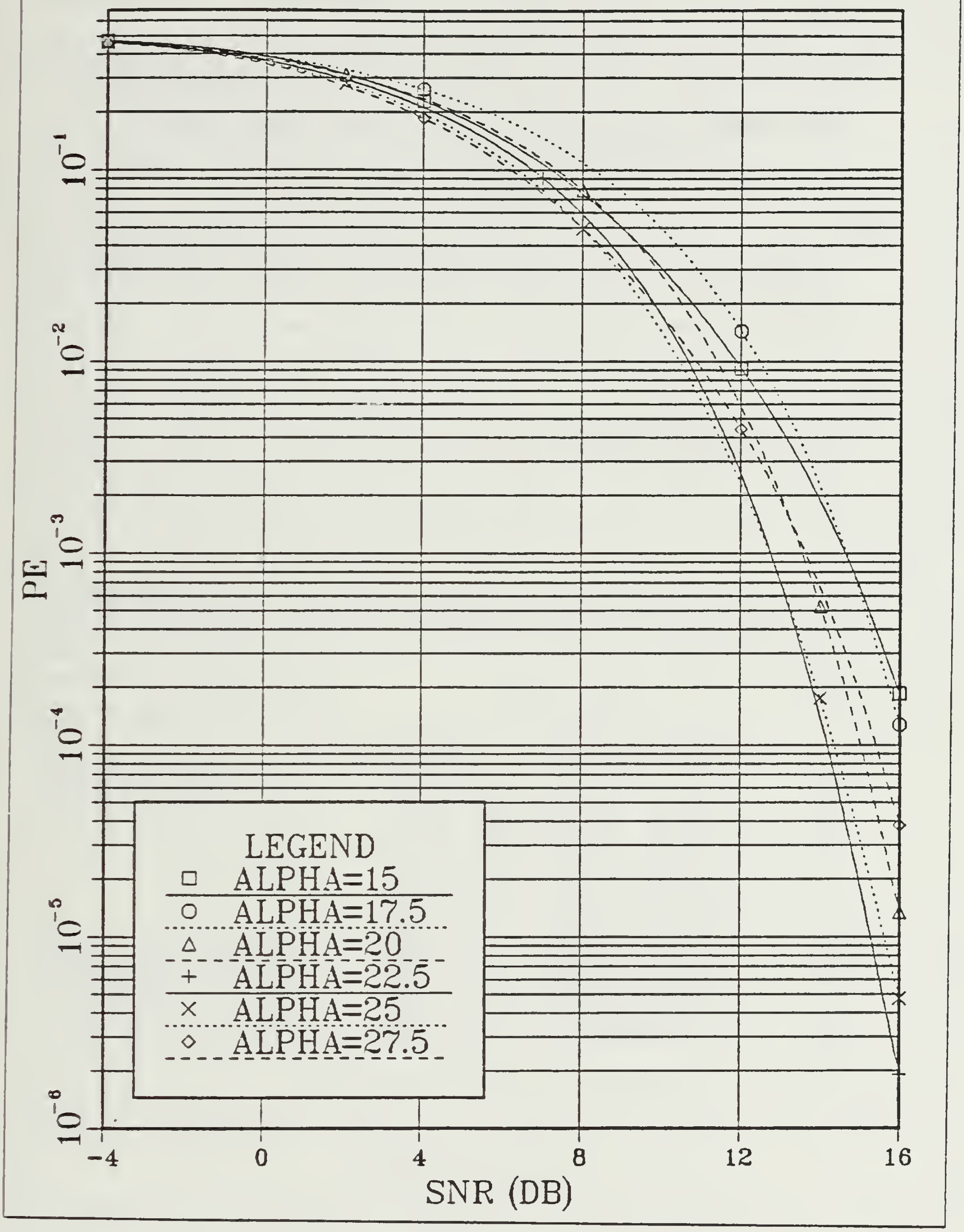

Figure 4.4 PE Versus SNR of 8-DPSK DBDR for Various Values of α ($\varepsilon = 0°$).

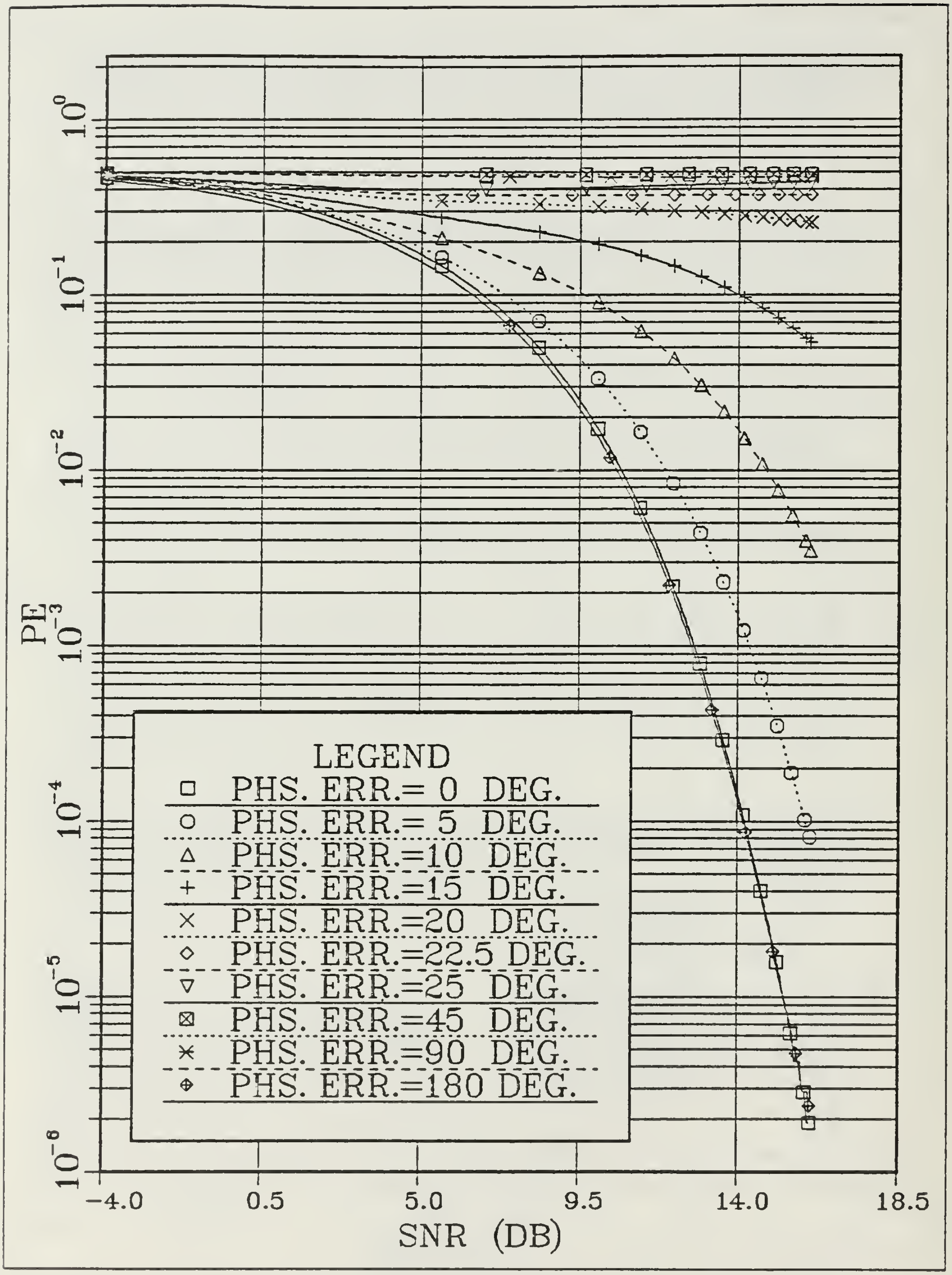

Figure 4.5 PE Versus SNR of 8-DPSK DBDR for Various Values of ε (α = 22.5°).

DBDRs on Figure 4.3, thus demonstrating the performance penalty of the latter receiver. While usage of 8-DPSK provides good bandwidth efficiency (same as 8-PSK) and relatively good noise immunity, the BER of 8-PSK is lower than that of 8-DPSK for $\varepsilon = 0°$, as expected

The value of using 8-DPSK versus 8-PSK can be appreciated more fully when comparing the performance plots of Figures 4.1 and 4.5. These show that if the receiver has a local oscillator phase error restricted to $0 \leq \varepsilon \leq 66.5°$, for similar SNR values, the BER of the 8-DPSK receiver is lower than that of the 8-PSK receiver. Since such phase errors can be typically expected to be small, this is a significant advantage. However it must be pointed out that for phase errors of more than 10°, both receivers exhibit severely degraded performance that would make both systems unsuitable for operation in a practical system. The 8-DPSK receiver does not exhibit complete insensitivity to phase errors as encountered in phase measurement receivers. Only when a complete phase revesal takes place (i.e., $\varepsilon = 180°$) is the performance of the 8-DPSK DBDR unaffected, while the 8-PSK receiver produces a nearly useless output.

The angle α set to 22.5° is optimum in minimum BER sense for the 8-PSK DBDR. The variations in PE versus α are shown in Figure 4.6, for $\varepsilon = 0°$ for 8-PSK, and a similar plot of PE versus α is plotted for various values of SNR and $\varepsilon = 0°$ on Figure 4.7 for the 8-DSPK case.

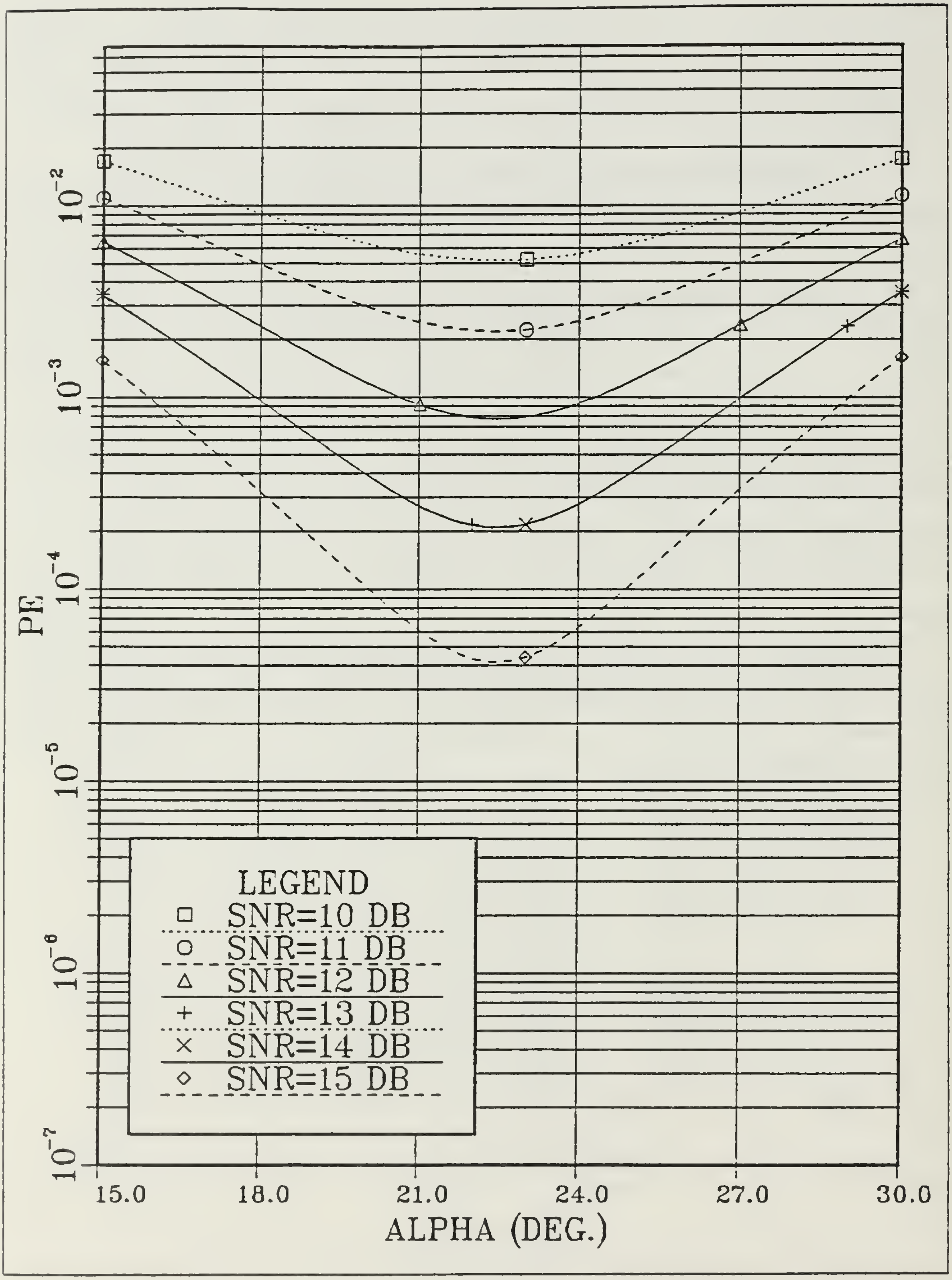

Figure 4.6 PE Versus α of 8-PSK DBDR for Various Values of SNR ($\varepsilon = 0°$).

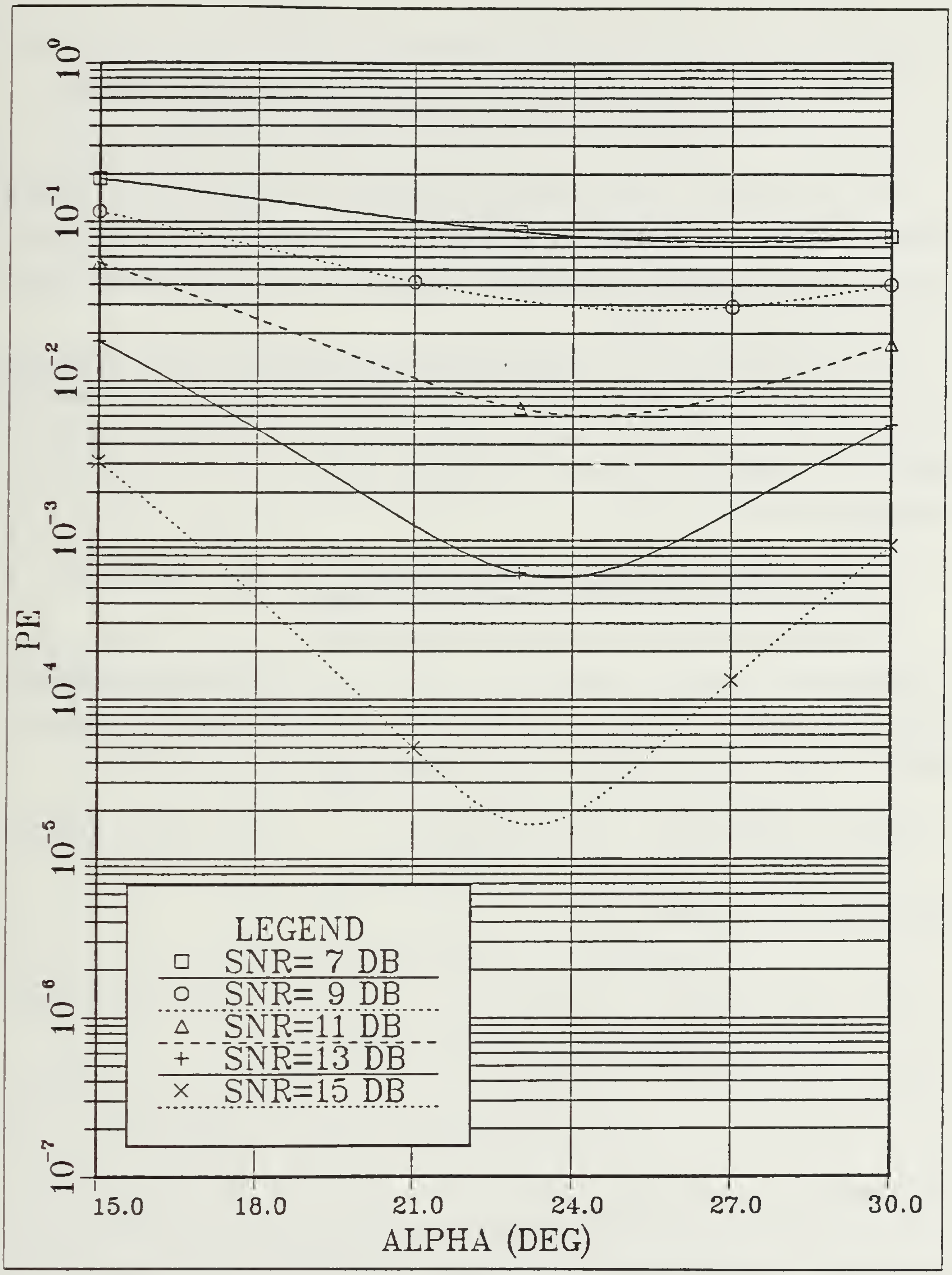

Figure 4.7 PE Versus α of 8-DPSK DBDR for Various Values of SNR ($\varepsilon = 0°$).

V. PERFORMANCE ANALYSES OF 16-PSK AND 16-DPSK DBDR'S WITH LOCAL OSCILLATOR PHASE ERROR

In this chapter, DBDRs will be analyzed for 16-PSK and 16-DPSK communication systems operating in the presence of a receiver local oscillator phase error. The analysis procedure will be similar to that carried out in the previous chapter for 8-PSK and 8-DPSK systems.

The performance of 16-PSK DBDR's has been investigated and the methodology for carrying out the performance analyses have been well-documented by Bukofzer [Ref. 8], In this research, these analyses will be extended to the 16-DPSK system as well as the 16-PSK system modified to operate in the presence of a receiver local oscillator phase error.

A. 16-PSK DBDR PERFORMANCE ANALYSIS IN THE PRESENCE OF RECEIVER LOCAL OSCILLATOR PHASE ERROR

The possible transmitted signals, which are represented as vectors in Figure 1.2.b, are mathematically given by Equation 4.1 with $i = 1,2,3,\dots,16$. The same notation introduced in Chapter IV will be used to represent the signals using the orthonormal set given by Equation 4.4. Therefore the transmitted signals are expressed as

$$s_i(t) = \varphi_1(t)\sqrt{E_s}\cos\theta_i(t) + \varphi_2(t)\sqrt{E_s}\sin\theta_i(t) \tag{5.2}$$

where $i = 1,2,\dots,16$, $0 \le t \le T_s$ and

$$\theta_i(t) = \begin{cases} \alpha + (i-1)\,\pi/8 & i = 1,5,9,13 \\ \beta + (i-2)\,\pi/8 & i = 2,6,10,14 \\ (i+1)\,\pi/8 - \beta & i = 3,7,11,15 \\ i\,\pi/8 - \alpha & i = 4,8,12,16 \end{cases} \tag{5.3}$$

with α and β as shown on Figure 1.2.b

Assuming all signals are equally likely to be transmitted, for the 16-PSK case Equation 4.7 can be written as

$$\Pr\{\text{ bit correct }\} = 1/16 \sum_{i=1}^{16} \Pr\{\text{ bit correct } / s_i(t) \text{ transmitted }\} \tag{5.4}$$

The notation to denote the four bits that make up a data state has been introduced in Chapter I. Also the notation introduced by Equation 4.13 is used here, namely

$$\Pr\{\text{ bit correct } / s_i(t)\} = \Pr\{\text{ bit correct } / s_i(t) \text{ transmitted }\} \quad i=1,2..,16 \tag{5.5}$$

From Figure 1.2.b, the conditional probabilities for the correct reception of the LB, MBL, MBR, RB are given by

$$\Pr\{\text{ LB correct } / s_i(t)\} = \begin{cases} \Pr\{ r_{1,i} > 0 / s_i(t)\} & i=1,2,3,4,13,14,15,16 \\ \Pr\{ r_{1,i} < 0 / s_i(t)\} & i=5,6,7,8,9,10,11,12 \end{cases} \tag{5.6}$$

$$\Pr\{\text{ MBL correct } / s_i(t)\} = \begin{cases} \Pr\{ r_{1,i}^2 - r_{2,i}^2 > 0 / s_i(t)\} & i=1,2,7,8,9,10,15,16 \\ \Pr\{ r_{1,i}^2 - r_{2,i}^2 < 0 / s_i(t)\} & i=3,4,5,6,11,12,13,14 \end{cases} \tag{5.7}$$

$$\Pr\{\text{ MBR correct } / s_i(t)\} = \begin{cases} \Pr\{ V_{Th} > 0 / s_i(t)\} & i=1,4,5,8,9,12,13,16 \\ \Pr\{ V_{Th} < 0 / s_i(t)\} & i=2,3,6,7,10,11,14,15 \end{cases} \tag{5.8}$$

$$\Pr\{\text{ RB correct } / s_i(t)\} = \begin{cases} \Pr\{ r_{2,i} > 0 / s_i(t)\} & i=1,2,3,4,5,6,7,8 \\ \Pr\{ r_{2,i} < 0 / s_i(t)\} & i=9,10,11,12,13,14,15,16 \end{cases} \tag{5.9}$$

where the $r_{k,i}$ are given by Equation 4.9 and

$$V_{Th} = (r_{1,i}^2 - r_{2,i}^2)^2 - 4 \, (r_{1,i} \, r_{2,i})^2 \,, \qquad i=1,2,3,....,16 \tag{5.10}$$

The analyses methods which have been given in Chapter IV, can be applied to determining the conditional error probabilities for LB, MBL, RB. For MBR, the analysis procedure which has been used by Bukofzer [Ref. 8], will be utilized. In order to define the conditional probability sets of Equation 5.8, additional steps must be completed as demonstrated in the sequel. If $s_i(t)$ transmitted, the MBR is correctly recovered if for i=1,4,5,8,9,12,13,16, $V_{Th} > 0$ and if for i=2,3,6,7,10,11,13,14, $V_{Th} < 0$. Even though the $r_{n,i}$ are conditional Gaussian random variables, because of the highly nonlinear form of Equation 5.10, a more productive approach to the determination of the conditional p.d.f. of V_{Th} has been introduced (see Reference 8 for details) rather than using transformations of Gaussian random variables. By introducing the following notation, namely

$$r_{1,i} = \text{component of } r(t) \text{ along } \varphi_1(t) = \ell \cos \eta \tag{5.11}$$

$$r_{2,i} = \text{component of } r(t) \text{ along } \varphi_2(t) = \ell \sin \eta \tag{5.12}$$

it can be demonstrated that

$$V_{Th} = \ell^4 \cos 4\eta \tag{5.13}$$

where

$$\ell = \text{length of the vector} = \sqrt{(r_{1,i})^2 + (r_{2,i})^2} \tag{5.14}$$

$$\eta = \text{vector angle that with respect to the positive x axis} \tag{5.15}$$

$$= \arctan\ (r_{2,i} / r_{1,i})$$

Since $\ell^4 > 0$, it is clear that the test involving V_{Th} being greater than or less than zero is now equivalent to the test involving $\cos 4\eta$ in the same manner. This implies that the p.d.f. of η must be obtained which, as shown in Reference 8, leads to integrals that cannot be evaluated in closed form and therefore must be evaluated via numerical integration.

By utilizing the analysis procedure from Reference 8 and a methodology introduced in the previous chapter for the 8-PSK case, the conditional probabilities for the correct reception of a bit in the presence of receiver local oscillator phase error is given in shorthand form by the following expressions

$$\Pr\{\text{ LB correct } / s_i(t) \} = 1 - Q\{ \sqrt{2 E_s/N_0}\, f(\alpha,\beta,\varepsilon) \}, \quad i=1,2,3...16 \tag{5.16}$$

where

$$f(\alpha.\beta,\varepsilon) = \begin{cases} \cos(\alpha-\varepsilon) & i=1,9 \\ \cos(\beta-\varepsilon) & i=2,10 \\ \sin(\beta+\varepsilon) & i=3,11 \\ \sin(\alpha+\varepsilon) & i=4,12 \\ \sin(\beta-\varepsilon) & i=5,13 \\ \sin(\alpha-\varepsilon) & i=6,14 \\ \cos(\beta+\varepsilon) & i=7,15 \\ \cos(\alpha+\varepsilon) & i=8,16 \end{cases} \qquad (5.17)$$

and α and β are shown on Figure 1.2.b. Furthermore

$$\Pr\{\text{ MBL correct } / s_i(t)\} = Q\{-\surd\, E_s/N_0\,(\cos\gamma+\sin\gamma)\,.\,Q\{-\surd E_s/N_0\,(\cos\gamma-\sin\gamma)\,\} + \\ + Q\{\surd E_s/N_0\,(\cos\gamma-\sin\gamma)\,\}.\ Q\{\surd E_s/N_0\,(\cos\gamma+\sin\gamma)\,\} \qquad (5.18)$$

where

$$\gamma = \begin{cases} (\alpha-\varepsilon) & i=1,5,9,13 \\ (\beta-\varepsilon) & i=2,6,10,14 \end{cases} \qquad (5.19)$$

or

$$\Pr\{\text{ MBL correct } / s_i(t)\} = 1 - Q\{-\surd E_s/N_0\,(\cos\gamma+\sin\gamma)\,.\,Q\{-\surd E_s/N_0\,(\cos\gamma-\sin\gamma)\,\} + \\ + Q\{\surd E_s/N_0\,(\cos\gamma+\sin\gamma)\}\,.\ Q\{\surd E_s/N_0\,(\cos\gamma-\sin\gamma)\} \qquad (5.20)$$

where

$$\gamma = \begin{cases} (\beta-\varepsilon) & i=3,7,11,15 \\ (\alpha-\varepsilon) & i=4,8,12,16 \end{cases} \qquad (5.21)$$

For the calculations involving the conditional probability for the MBR, the joint p.d.f. of the random variables η and ℓ has been determined by Bukofzer [Ref. 8] namely

$$f_{\ell,\eta/s_i}(L\,,\,\mathbf{H}) = |L|\, f_{r_{1,i},r_{2,i}/s_i}(L\cos\mathbf{H}\,,\,L\sin\mathbf{H}) + \\ + |L|\, f_{r_{1,i},r_{2,i}/s_i}(-L\cos\mathbf{H}\,,\,-L\sin\mathbf{H}) \qquad (5.22)$$

which must be integrated to produce the marginal p.d.f. of η. This results in

$$\begin{aligned} f_{\eta/s_i}(\mathbf{H}/s_i) = {} & 1/2\exp(-E_s/N_0) + \{1 + \sqrt{4E_s/\pi N_0}\,\cdot \\ & \cdot\,\{\exp[E_s/N_0\cos^2(\mathbf{H}-\xi_i)]\cos(\mathbf{H}-\xi_i)\,.\,Q[-\sqrt{2E_s/N_0}\,(\cos(\mathbf{H}-\xi_i)]\,\} \end{aligned} \tag{5.23}$$

where

$$\xi_i = \theta_i(t) - \varepsilon \qquad i=1,2....16 \tag{5.24}$$

where $\theta_i(t)$ is given by Equation 5.3 Therefore, the probability that $V_{Th} > 0$ involves computing the probability that $\cos 4\eta > 0$. Observe that $\cos 4\eta > 0$ for $-\pi/8 < \eta < \pi/8$, $3\pi/8 < \eta < 5\pi/8$, $7\pi/8 < \eta < 9\pi/8$, and $11\pi/8 < \eta < 13\pi/8$. Therefore the calculation of the conditional probability of the MBR being correct given that $s_1(t)$ was transmitted can be calculated by integrating the conditional p.d.f of η over the appropriate regions, namely

$$\begin{aligned} & \Pr\{\text{ MBR correct } / s_1(t)\} = \Pr\{\cos 4\eta > 0 / s_1(t)\} = \\ & = \int_{-\pi/8}^{\pi/8} f_{\eta/s_1}[\mathbf{H}/s_1]\,d\mathbf{H} + \int_{3\pi/8}^{5\pi/8} f_{\eta/s_1}[\mathbf{H}/s_1]\,d\mathbf{H} + \int_{7\pi/8}^{9\pi/8} f_{\eta/s_1}[\mathbf{H}/s_1]\,d\mathbf{H} + \\ & + \int_{11\pi/8}^{13\pi/8} f_{\eta/s_1}[\mathbf{H}/s_1]\,d\mathbf{H} \\ & = \int_{-\pi/8}^{\pi/8} \{ f_{\eta/s_1}[\mathbf{H}/s_1] + f_{\eta/s_1}[(\mathbf{H}+\pi/2)/s_1] + f_{\eta/s_1}[(\mathbf{H}+\pi)/s_1] + \\ & \quad + f_{\eta/s_1}[(\mathbf{H}+3\pi/2)/s_1]\}\,d\mathbf{H} \end{aligned} \tag{5.25}$$

By utilizing the appropriate form for the conditional p.d.f. of η when $s_i(t)$ is assumed transmitted, results in

$$\begin{aligned} & \Pr\{\text{ MBR correct } / s_i(t)\} = 1/2\exp(-E_s/N_0) + \\ & + \sqrt{E_s/\pi N_0}\int_{-\pi/8}^{\pi/8}\{\exp[-E_s/N_0\sin^2(\mathbf{H}-\lambda)]\cos(\mathbf{H}-\lambda)\,.\,\{Q[-\sqrt{2E_s/N_0}\,(\cos(\mathbf{H}-\lambda)] - \end{aligned}$$

$$- Q[\sqrt{2\,E_s/N_0}\,(\cos(H+\lambda)]\,\} + \{\,\exp[-E_s/N_0 \cos^2(H-\lambda)]\,\sin(H-\lambda)\,Q[-\sqrt{2\,E_s/N_0}\,\,(\sin(H-\lambda)] -$$
$$- Q[\sqrt{2\,E_s/N_0}\,(\sin(H+\lambda)\,]\,\}\,\}\,dH \qquad (5.26)$$

where

$$\lambda = \begin{cases} \alpha+\varepsilon & i=4,8,12,16 \\ \alpha-\varepsilon & i=1,5,9,13 \\ \beta-\varepsilon & i=2,6,10,14 \\ \beta+\varepsilon & i=3,7,11,15 \end{cases} \qquad (5.27)$$

Finally, it can be seen that

$$\Pr\{\text{ RB correct } / \ s_i(t)\,\} = \Pr\{\text{ LB correct } / \ s_i(t)\,\} \qquad (5.28)$$

so that by using Equations 5.4 and 5.16-5.28 for correct reception probabilities of the individual bits, the overall BER can be obtained by using

$$\Pr\{\text{ bit in error }\} = 1 - \Pr\{\text{ bit correct }\} \qquad (5.29)$$

This results in the (unconditional) probabilities

$$\begin{aligned} \Pr\{\text{ LB in error }\} = \Pr\{\text{ RB in error }\} \\ &= 1/8\,\{\,Q[\,\sqrt{2\,E_s/N_0}\,\ \cos(\alpha-\varepsilon)] + Q[\,\sqrt{2\,E_s/N_0}\,\ \cos(\beta-\varepsilon)] + \\ &+ Q[\,\sqrt{2\,E_s/N_0}\,\ \sin(\beta+\varepsilon)] + Q[\sqrt{2\,E_s/N_0}\,\ \sin(\alpha+\varepsilon)] + \\ &+ Q[\,\sqrt{2\,E_s/N_0}\,\ \sin(\beta-\varepsilon)] + Q[\sqrt{2\,E_s/N_0}\,\ \cos(\beta+\varepsilon)] + \\ &+ Q\,[\sqrt{2\,E_s/N_0}\,\ \cos(\alpha+\varepsilon)] + \ Q\,[\sqrt{2\,E_s/N_0}\,\ \sin(\alpha+\varepsilon)]\} \qquad (5.30) \end{aligned}$$

and

$$\begin{aligned} \Pr\{\text{ MBL in error }\} &= 1/2 + 1/4\,[\ \ Q\,\{-\sqrt{E_s/N_0}\,[\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)]\} + \\ &+ Q\,\{-\sqrt{E_s/N_0}\,[\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)]\} + Q\,\{-\sqrt{E_s/N_0}\,[\cos(\beta-\varepsilon) + \sin(\beta-\varepsilon)]\} + \\ &+ Q\,\{-\sqrt{E_s/N_0}\,[\cos(\beta-\varepsilon) - \sin(\beta-\varepsilon)]\} - Q\,\{-\sqrt{E_s/N_0}\,[\cos(\beta+\varepsilon) + \sin(\beta+\varepsilon)]\} - \\ &- Q\,\{-\sqrt{E_s/N_0}\,[\cos(\beta+\varepsilon) - \sin(\beta+\varepsilon)]\} - Q\,\{-\sqrt{E_s/N_0}\,[\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)]\} - \end{aligned}$$

$$\begin{aligned}
&- Q \{-\sqrt{E_s/N_0}\, [\cos(\alpha+\varepsilon) - \sin(\alpha+\varepsilon)]\} \,] + 1/2 \,[\, Q \{-\sqrt{E_s/N_0}\, [\cos(\alpha+\varepsilon) + \sin(\alpha+\varepsilon)]\} \,. \\
&. \, Q \{-\sqrt{E_s/N_0}\, [\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)]\} + Q \{-\sqrt{E_s/N_0}\, [\cos(\beta+\varepsilon) + \sin(\beta+\varepsilon)]\} \,. \\
&. \, Q \{-\sqrt{E_s/N_0}\, [\cos(\beta+\varepsilon) - \sin(\beta+\varepsilon)]\} - Q \{-\sqrt{E_s/N_0}\, [\cos(\alpha-\varepsilon) + \sin(\alpha-\varepsilon)]\} \,. \\
&. \, Q \{-\sqrt{E_s/N_0}\, [\cos(\alpha-\varepsilon) - \sin(\alpha-\varepsilon)]\} - Q \{-\sqrt{E_s/N_0}\, [\cos(\beta-\varepsilon) + \sin(\beta-\varepsilon)]\} \,. \\
&. \, Q \{-\sqrt{E_s/N_0}\, [\cos(\beta-\varepsilon) - \sin(\beta-\varepsilon)]\} \,]
\end{aligned} \tag{5.31}$$

For the MBR the result appears in integral form. That is

$$\begin{aligned}
\Pr\{ \text{ MBR in error } \} = 1 - 1/2 \sqrt{E_s/\pi N_0} \int_{-\pi/8}^{\pi/8} &\{ \exp\{-E_s/N_0 \sin^2[H-(\alpha-\varepsilon)]\} \cos[H-(\alpha-\varepsilon)]. \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \cos(H-(\alpha-\varepsilon))]\} &+ \exp\{-E_s/N_0 \cos^2[H-(\alpha-\varepsilon)]\} \sin[H-(\alpha-\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \sin(H-(\alpha-\varepsilon))]\} &+ \exp\{-E_s/N_0 \sin^2[H-(\beta-\varepsilon)]\} \cos[H-(\beta-\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \cos(H-(\beta-\varepsilon))]\} &+ \exp\{-E_s/N_0 \cos^2[H-(\beta-\varepsilon)]\} \sin[H-(\beta-\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \sin(H-(\beta-\varepsilon))]\} &- \exp\{-E_s/N_0 \sin^2[H+(\beta+\varepsilon)]\} \cos[H+(\beta+\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \cos(H+(\beta+\varepsilon))]\} &- \exp\{-E_s/N_0 \cos^2[H+(\beta+\varepsilon)]\} \sin[H+(\beta+\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \sin(H+(\alpha+\varepsilon))]\} &+ \exp\{-E_s/N_0 \sin^2[H+(\alpha+\varepsilon)]\} \cos[H+(\alpha+\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \cos(H+(\alpha+\varepsilon)]\} &+ \exp\{-E_s/N_0 \cos^2[H+(\alpha+\varepsilon)]\} \sin[H+(\alpha+\varepsilon)] \\
. \{ 1/2 - Q[-\sqrt{E_s/N_0} \sin(H+(\alpha+\varepsilon))]\} &\} \, dH
\end{aligned} \tag{5.32}$$

The overall BER can now be determined from Equations 5.30,5.31 and 5.32 by evaluating the final result for PE, the bit error probability, namely

$$\begin{aligned}
PE = 1/4 \{ &\Pr\{ \text{ LB in error } \} + \Pr\{ \text{ MBL in error } \} \\
&+ \Pr\{ \text{ MBR in error } \} + \Pr\{ \text{ RB in error } \} \}
\end{aligned} \tag{5.33}$$

It can be seen that Equation 5.33 consists of many terms that by themselves do not yield as much insight as the computed values of PE presented in plotted form. Therefore plots of BER vs. SNR for various values of α and ε, are given as Figures 5.1 and 5.2. While α and β can be set independently, from an optimization point of view, $\alpha+\beta=\pi/4$ has been shown to produce the lowest BER for given values of SNR when $\varepsilon=0°$ [Ref. 8]

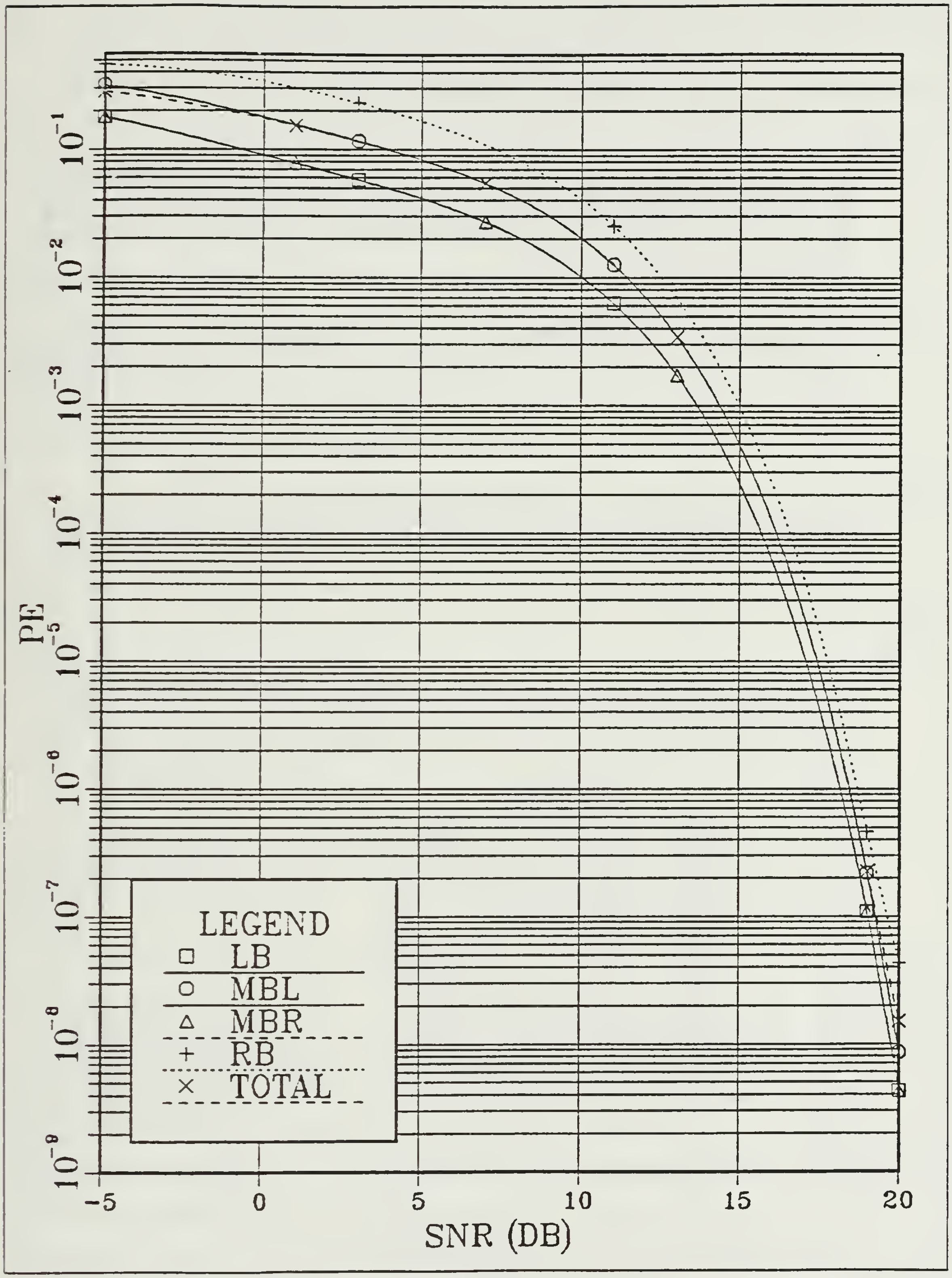

Figure 5.1 PE Versus SNR for 16-PSK DBDR ($\alpha = 11.25^\circ$, $\varepsilon = 0^\circ$).

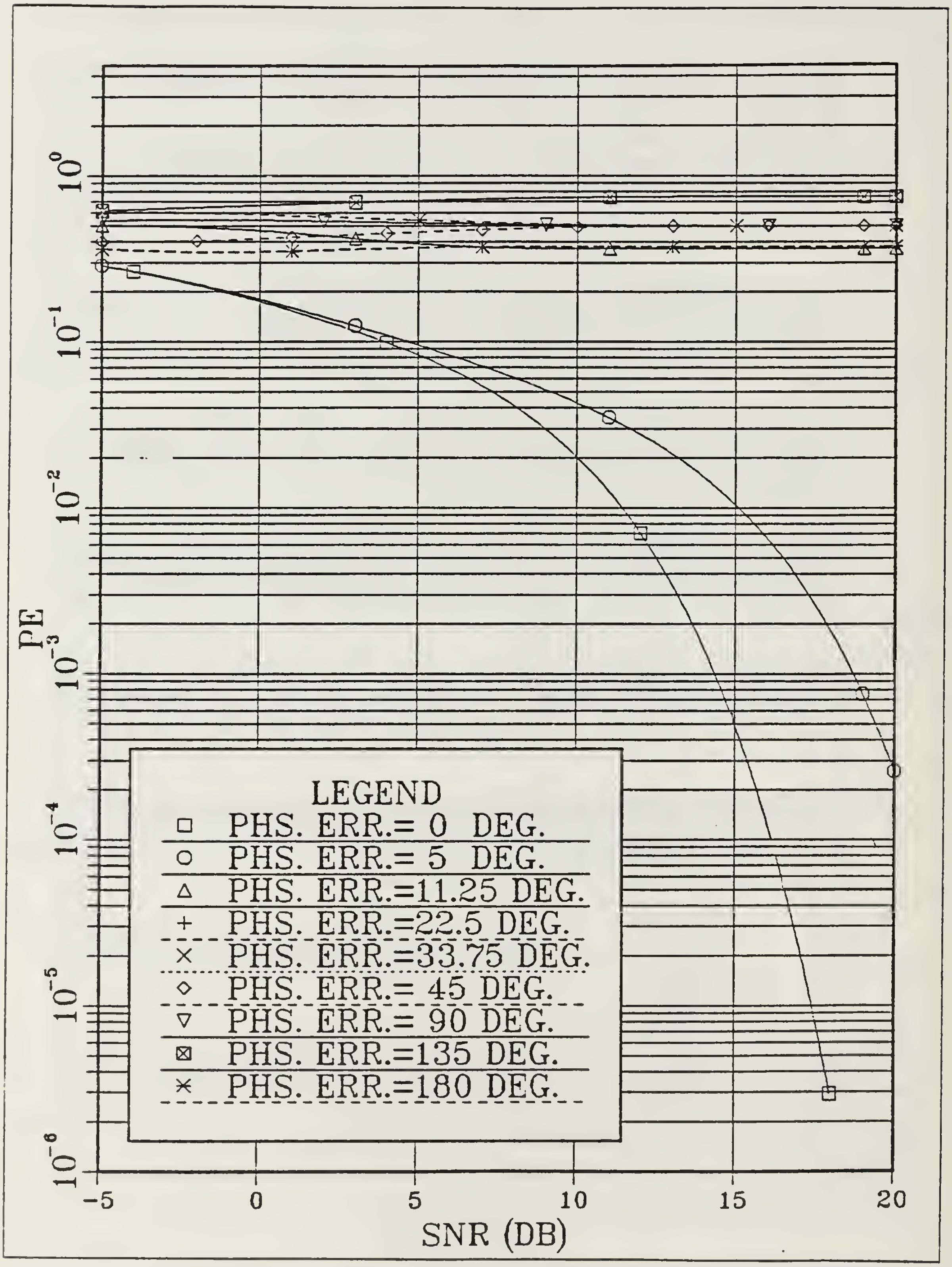

Figure 5.2 PE Versus SNR for 16-PSK DBDR for Various Values of ε (α = 11.25°).

B. 16-DPSK DBDR PERFORMANCE ANALYSIS IN THE PRESENCE OF RECEIVER LOCAL OSCILLATOR PHASE ERROR

Similar analyses on DBDR performance have been carried out for 16-DPSK (see Chapter IV.B), so that the notation introduced previously will be used here. That is,

$$Ck = \Pr\{\ k \text{ correct}\ \} \quad , \quad Ek = \Pr\{\ k \text{ in error}\ \} \tag{5.34}$$

where now k=1 for LB, k= 2 for MBL, k=3 for MBR and k= 4 for RB.

In order to evaluate Pr { LB correct } with differential decoding, we have to evaluate

$$\begin{aligned}\Pr\{\ \text{LB correct}\ \} = 1/2\ [\ &\Pr\{\ \text{LB correct} / \text{LB}=0\ \} + \\ &+ \{\ \Pr\{\ \text{LB correct} / \text{LB}=1\ \}\ ]\end{aligned} \tag{5.35}$$

where

$$\begin{aligned}&\Pr\{\ \text{LB correct} / \text{LB}=0\ \} = \\ &= \sum_{i=0}^{1} \sum_{j=0}^{1} \sum_{k=0}^{1} \Pr\{\ \text{LB correct} / \text{LB}=0, \text{MBL}=i, \text{MBR}=j, \text{RB}=k\ \} \times \\ &\qquad \times \Pr\{\ \text{MBL}=i, \text{MBR}=j, \text{RB}=k\ \}\end{aligned} \tag{5.36}$$

where for i, j, k= 0, 1

$$\Pr\{\ \text{MBL}=i, \text{MBR}=j, \text{RB}=k\ \} = 1/8 \tag{5.37}$$

In Chapter IV, it has been explained that with DPSK modulation in conjuction with bit direct detection methods, certain bit errors in the past and presently recovered symbol states do not result in errors in the recovered data states. Such bit errors in successive symbol states must be considered, because, even though symbol errors may occur, the delivered data may still be correct. Therefore, these errors have to be accounted for and considered in the computation of the probability of a bit being correct. Figure 5.3 shows one example of the situation being discussed. That is, the data state (0,0,0,0) is assumed to occur and the symbol state (0,0,0,0) is assumed to

```
#######################################################################
       CORRECT RECEPTION COMBINATIONS IN PRESENCE OF ERROR
#######################################################################
    INPUT     PREVIOUS      PRESENT       PREVIOUS    PRESENT      OUTPUT
            TRANSMISSION TRANSMISSION   RECEPTION   RECEPTION
-----------------------------------------------------------------------
 0 0 0 0       0 0 0 0      0 0 0 0      0 0 0 0      0 0 0 0    0 0 0 0
                                         0 0 0 1      0 0 0 1    0 0 0 0
                                         0 0 1 0      0 0 1 0    0 0 0 0
                                         0 0 1 1      0 0 1 1    0 0 0 0
                                         0 1 0 0      0 1 0 0    0 0 0 0
                                         0 1 0 1      0 1 0 1    0 0 0 0
                                         0 1 1 0      0 1 1 0    0 0 0 0
                                         0 1 1 1      0 1 1 1    0 0 0 0
                                         1 0 0 0      1 0 0 0    0 0 0 0
                                         1 0 0 1      1 0 0 1    0 0 0 0
                                         1 0 1 0      1 0 1 0    0 0 0 0
                                         1 0 1 1      1 0 1 1    0 0 0 0
                                         1 1 0 0      1 1 0 0    0 0 0 0
                                         1 1 0 1      1 1 0 1    0 0 0 0
                                         1 1 1 0      1 1 1 0    0 0 0 0
                                         1 1 1 1      1 1 1 1    0 0 0 0
                                         0 0 0 0      1 0 0 0    0 0 1 0
                                         0 0 0 1      0 1 1 0    0 0 1 0
                                         0 0 1 0      1 1 0 0    0 0 1 0
                                         0 0 1 1      0 1 1 1    0 0 1 0
                                         0 1 0 0      1 0 1 0    0 0 1 0
                                         0 1 0 1      0 1 0 0    0 0 1 0
                                         0 1 1 0      0 0 1 0    0 0 1 0
                                         0 1 1 1      0 0 0 0    0 0 1 0
                                         1 0 0 0      1 1 1 1    0 0 1 0
                                         1 0 0 1      0 0 0 1    0 0 1 0
                                         1 0 1 0      1 1 1 0    0 0 1 0
                                         1 0 1 1      0 1 0 1    0 0 1 0
                                         1 1 0 0      1 1 0 1    0 0 1 0
                                         1 1 0 1      0 0 1 1    0 0 1 0
                                         1 1 1 0      1 0 0 1    0 0 1 0
                                         1 1 1 1      1 0 1 1    0 0 1 0
                                         0 0 0 0      1 1 0 1    0 1 0 0
                                         0 0 0 1      1 0 1 0    0 1 0 0
                                         0 0 1 0      1 0 0 1    0 1 0 0
                                         0 0 1 1      0 0 1 0    0 1 0 0
                                         0 1 0 0      1 1 1 1    0 1 0 0
                                         0 1 0 1      1 0 0 0    0 1 0 0
                                         0 1 1 0      1 1 1 0    0 1 0 0
                                         0 1 1 1      1 1 0 0    0 1 0 0
                                         1 0 0 0      0 0 1 1    0 1 0 0
                                         1 0 0 1      0 1 0 0    0 1 0 0
                                         1 0 1 0      1 0 1 1    0 1 0 0
                                         1 0 1 1      0 0 0 0    0 1 0 0
                                         1 1 0 0      0 0 0 1    0 1 0 0
                                         1 1 0 1      0 1 1 0    0 1 0 0
                                         1 1 1 0      0 1 0 1    0 1 0 0
                                         1 1 1 1      0 1 1 1    0 1 0 0
                                         0 0 0 0      1 1 0 0    0 1 1 0
                                         0 0 0 1      0 1 0 0    0 1 1 0
                                         0 0 1 0      1 1 1 0    0 1 1 0
                                         0 0 1 1      0 1 1 0    0 1 1 0
                                         0 1 0 0      1 0 0 0    0 1 1 0
                                         0 1 0 1      0 0 0 0    0 1 1 0
                                         0 1 1 0      1 0 1 0    0 1 1 0
                                         0 1 1 1      0 0 1 0    0 1 1 0
                                         1 0 0 0      1 1 0 1    0 1 1 0
                                         1 0 0 1      0 1 0 1    0 1 1 0
                                         1 0 1 0      1 1 1 1    0 1 1 0
                                         1 0 1 1      0 1 1 1    0 1 1 0
                                         1 1 0 0      1 0 0 1    0 1 1 0
                                         1 1 0 1      0 0 0 1    0 1 1 0
```

Figure 5.3 Example of Error Patterns for 16-DPSK System Where Data State, Previous Transmission, and Present Transmission are 0000 with RB Correctly Recovered.

INPUT	PREVIOUS TRANSMISSION	PRESENT TRANSMISSION	PREVIOUS RECEPTION	PRESENT RECEPTION	OUTPUT
			1 1 1 0	1 0 1 1	0 1 1 0
			1 1 1 1	0 0 1 1	0 1 1 0
			0 0 0 0	0 1 0 0	1 0 0 0
			0 0 0 1	0 0 1 1	1 0 0 0
			0 0 1 0	0 0 0 0	1 0 0 0
			0 0 1 1	1 0 1 1	1 0 0 0
			0 1 0 0	0 1 1 0	1 0 0 0
			0 1 0 1	0 0 0 1	1 0 0 0
			0 1 1 0	0 1 1 1	1 0 0 0
			0 1 1 1	0 1 0 1	1 0 0 0
			1 0 0 0	1 0 1 0	1 0 0 0
			1 0 0 1	1 1 0 1	1 0 0 0
			1 0 1 0	0 0 1 0	1 0 0 0
			1 0 1 1	1 0 0 1	1 0 0 0
			1 1 0 0	1 0 0 0	1 0 0 0
			1 1 0 1	1 1 1 1	1 0 0 0
			1 1 1 0	1 1 0 0	1 0 0 0
			1 1 1 1	1 1 1 0	1 0 0 0
			0 0 0 0	0 1 0 1	1 0 1 0
			0 0 0 1	1 1 0 1	1 0 1 0
			0 0 1 0	0 1 1 1	1 0 1 0
			0 0 1 1	1 1 1 1	1 0 1 0
			0 1 0 0	0 0 0 1	1 0 1 0
			0 1 0 1	1 0 0 1	1 0 1 0
			0 1 1 0	0 0 1 1	1 0 1 0
			0 1 1 1	1 0 1 1	1 0 1 0
			1 0 0 0	0 1 0 0	1 0 1 0
			1 0 0 1	1 1 0 0	1 0 1 0
			1 0 1 0	0 1 1 0	1 0 1 0
			1 0 1 1	1 1 1 0	1 0 1 0
			1 1 0 0	0 0 0 0	1 0 1 0
			1 1 0 1	1 0 0 0	1 0 1 0
			1 1 1 0	0 0 1 0	1 0 1 0
			1 1 1 1	1 0 1 0	1 0 1 0
			0 0 0 0	1 0 0 1	1 1 0 0
			0 0 0 1	1 0 0 0	1 1 0 0
			0 0 1 0	1 0 1 1	1 1 0 0
			0 0 1 1	1 0 1 0	1 1 0 0
			0 1 0 0	1 1 0 1	1 1 0 0
			0 1 0 1	1 1 0 0	1 1 0 0
			0 1 1 0	1 1 1 1	1 1 0 0
			0 1 1 1	1 1 1 0	1 1 0 0
			1 0 0 0	0 0 0 1	1 1 0 0
			1 0 0 1	0 0 0 0	1 1 0 0
			1 0 1 0	0 0 1 1	1 1 0 0
			1 0 1 1	0 0 1 0	1 1 0 0
			1 1 0 0	0 1 0 1	1 1 0 0
			1 1 0 1	0 1 0 0	1 1 0 0
			1 1 1 0	0 1 1 1	1 1 0 0
			1 1 1 1	0 1 1 0	1 1 0 0
			0 0 0 0	0 0 0 1	1 1 1 0
			0 0 0 1	1 1 1 1	1 1 1 0
			0 0 1 0	0 1 0 1	1 1 1 0
			0 0 1 1	1 1 1 0	1 1 1 0
			0 1 0 0	0 0 1 1	1 1 1 0
			0 1 0 1	1 1 0 1	1 1 1 0
			0 1 1 0	1 0 1 1	1 1 1 0
			0 1 1 1	1 0 0 1	1 1 1 0
			1 0 0 0	0 1 1 0	1 1 1 0
			1 0 0 1	1 0 0 0	1 1 1 0
			1 0 1 0	0 1 1 1	1 1 1 0
			1 0 1 1	1 1 0 0	1 1 1 0
			1 1 0 0	0 1 0 0	1 1 1 0
			1 1 0 1	1 0 1 0	1 1 1 0
			1 1 1 0	0 0 0 0	1 1 1 0
			1 1 1 1	0 0 1 0	1 1 1 0

Figure 5.3 Continued.

have been previously transmitted. This means that according to the encoder matrix shown in Chapter II, the symbol state (0,0,0,0) must now be generated for transmission. Figure 5.3 shows the receptions of the symbol states which produce recovered data states in which the RB is correctly delivered. Observe that there are 128 such combinations which yield 0 for the RB. Therefore notation has to be introduced to account for error patterns of the above type, namely Pr{ RB correct, (g,h,p,q) / LB = m,MBL = i,MBR = j,RB = k }, where g,h,p,q = 0,1. The meaning of this expression is the probability of the RB being correctly recovered when the data state is (m,j,i,k) and the delivered data state is (g,h,p,q) where q = 0 in this case (otherwise the RB could not be correct due to the transmission of the RB = 0). Therefore

$$\Pr\{ \text{RB correct} / \text{LB}=0, \text{MBL}=0, \text{MBR}=0, \text{RB}=0 \} =$$
$$= \sum_{g=0}^{1} \sum_{h=0}^{1} \sum_{p=0}^{1} \Pr\{ \text{RB correct}, (g,h,p,0) / \text{LB}=0, \text{MBL}=0, \text{MBR}=0, \text{RB}=0 \} \quad (5.38)$$

Since each element of the symbol state is independently recovered, we see from Figure 5.3 that

$$\begin{aligned}
&\Pr\{ \text{RB correct},(0,0,0,0) / \text{LB}=0,\text{MBL}=0,\text{MBR}=0,\text{RB}=0 \} = C1^2\, C2^2\, C3^2\, C4^2 + \\
&+ C1^2\, C2^2\, E3^2\, C4^2 + C1^2\, C2^2\, E3^2\, E4^2 + C1^2\, E2^2\, C3^2\, E4^2 + C1^2\, E2^2\, E3^2\, C4^2 + \\
&+ C1^2\, E2^2\, E3^2\, E4^2 + E1^2 + C2^2\, C3^2\, C4^2 + E1^2\, C2^2\, C3^2\, E4^2 + E1^2\, C2^2\, E3^2\, C4^2 + \\
&+ E1^2\, C2^2\, E3^2\, E4^2 + E1^2\, E2^2\, C3^2\, C4^2 + E1^2\, E2^2\, E3^2\, C4^2 + E1^2\, E2^2\, E3^2\, E4^2 + \\
&+ C1^2\, C2^2\, C3^2\, E4^2 \qquad (5.39)
\end{aligned}$$

However, from Equations 5.34, and 5.30, it is shown that C1 = C4 and E1 = E4. Therefore,

$$\begin{aligned}
&\Pr\{ \text{RB correct}, (0,0,0,0) / \text{LB}=0, \text{MBL}=0, \text{MBR}=0, \text{RB}=0 \} = \\
&= C1^4\, C2^2\, C3^2 + 2\, C1^2\, C2^2\, C3^2\, E1^2 + C2^2\, C3^2\, E1^2\, E2^2 \\
&+ 2\, C1^4\, C2^2\, E3^2 + 2\, C1^2\, C2^2\, E1^2\, E3^2 + C1^4\, C3^2\, E2^2 + E1^4\, E2^2\, E3^2 \\
&+ 2\, C1^2\, C3^2\, E1^2\, E2^2 + C2^2\, E1^2\, E2^2\, E3^2 + C1^2\, E1^2\, E2^2\, E3^2 + \\
&+ C3^2\, E1^4\, E2^2 + C1^2\, C3^2\, E1^4 \qquad (5.40)
\end{aligned}$$

A similar expression must be obtained for other combinations of values of g,h and p as well as the condition that RB is 1. This produces a total of 256 possibilities of received symbol states that result in a correct RB. Computer generated outputs were used to identify the equivalent error patterns for other cases involving LB = 1, MBL = 1 and MBR = 1. These results are shown in Appendix C. Similar analyses must be carried out in order to determine the unconditional probabilities Pr{LB correct}, Pr{MBL correct} as well as Pr{MBR correct}. Such analyses demonstrate due to inherent symmetries that

$$\Pr\{\text{LB correct}\} = \Pr\{\text{MBL correct}\} = \Pr\{\text{MBR correct}\} = \Pr\{\text{RB correct}\} \equiv PE \tag{5.41}$$

so that using Equation 5.38 along with similar forms obtained for k = 1 and l = 1 yield

$$\begin{aligned} PE = {} & 1 - 1/2 \{ 2\,C1^4\,C2^2\,C3^2 + C1^4\,C2^2\,C3\,E3 + 3\,C1^4\,C2^2\,C3\,E2 + \\ & + 2\,C1^4\,C2\,C3\,E2\,E3 + 6\,C1^3\,C2^2\,C3^2\,E1 + 4\,C1^3\,C2^2\,C3\,E1\,E3 + \\ & + 12\,C1^3\,C2\,C3^2\,C3\,E1\,E2 + 8\,C1^3\,C2\,C3\,C3\,E1\,E2 + 6\,C1^3\,C2^2\,E1\,E3^2 + \\ & + 6\,C1^3\,C2^2\,C3^2\,E1 + C1^4\,C3\,E2^2\,E3 + 4\,C1^3\,C3\,E1\,E2^2\,E3 + \\ & + 2\,C1^4\,E1^2\,E3^2 + C1^4\,C2^2\,E3^2 + 3\,C1^4\,C2\,E2\,E3^2 + 2\,C2^2\,C3^2\,E1^4 + \\ & + C1^4\,C3^2\,E2^2 + 10\,C1^2\,C2^2\,C3^2\,E1^2 + 10\,C1^2\,C2^2\,E1^2\,E3^2 + \\ & + 10\,C1^2\,C3^2\,E1^2\,E2^2 + 8\,C1^2\,E1^2\,E2^2\,E3^2 + 18\,C1^2\,C2\,C3^2\,E1^2\,E2^2 + \\ & + 18\,C1^2\,C2\,E1^2\,E2\,E3^2 + 6\,C1^3\,E1\,E2^2\,E3^2 + 12\,C1^3\,C2\,E1\,E2\,E3^2 + \\ & + 12\,C1^2\,C2\,C3\,E1^2\,E2\,E3 + 6\,C1^2\,C2^2\,C3\,E1^2\,E3 + 5\,C1^2\,C3\,E1^2\,E2^2\,E3 + \\ & + 6\,C1\,C2^2\,C3^2\,E1^3 + 6\,C1\,C3^2\,E1^3\,E2^2 + 6\,C1\,C2^2\,E1^3\,E2^2 + C2^2\,E1^4\,E3^2 \\ & + 12\,C1\,C2\,C3^2\,E1^3\,E2 + 12\,C1\,C2\,E1^3\,E2\,E3^2 + 6\,C1\,E1^3\,E2_2\,E3^2 + \\ & + 8\,C1\,C2\,C3\,E1^3\,E2\,E3 + 4\,C1\,C3\,E1^3\,E2_2\,E3 + 4\,C1\,C2^2\,C3\,E1^3\,E3 + \\ & + C3^2\,E1^4\,E2^2 + 2\,E1^4\,E2^2\,E3^2 + C2^2\,C3\,E1^4\,E3 + C3\,E1^4\,E2^2\,E3 + \\ & + 3\,C2\,E1^4\,E2\,E3^2 + 2\,C2\,C3\,E1^4\,E2\,E3 + 2\,C2\,C3^2\,E1^4\,E2 \} \end{aligned} \tag{5.42}$$

In Figure 5.4, PE (given by Equation 5.42) is plotted for various values of α. Also Figure 5.5 includes BER for various values of ε when $\alpha = 11.25^\circ$

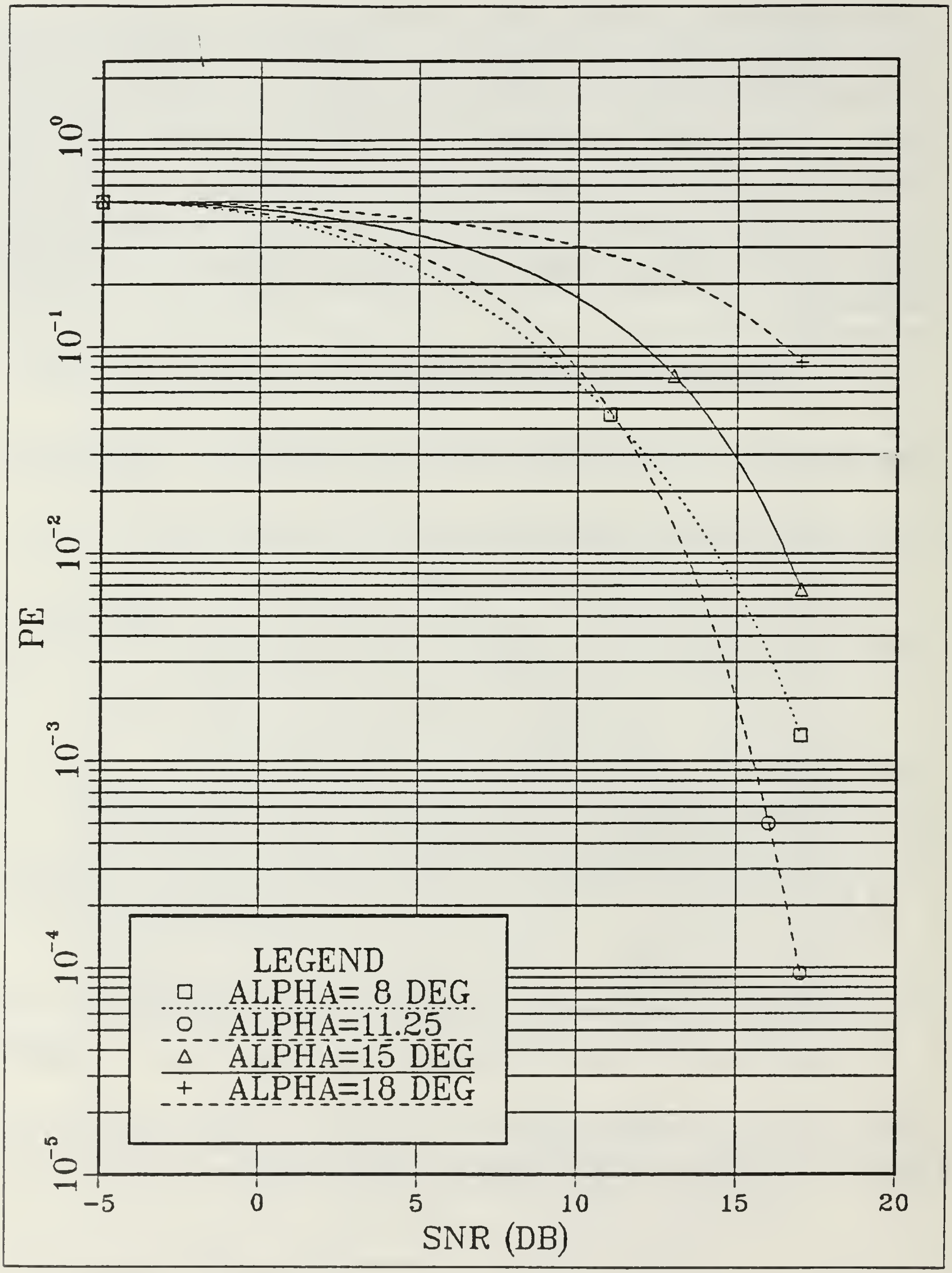

Figure 5.4 PE Versus SNR (dB) for 16-DPSK for Various Values of α ($\varepsilon = 0°$).

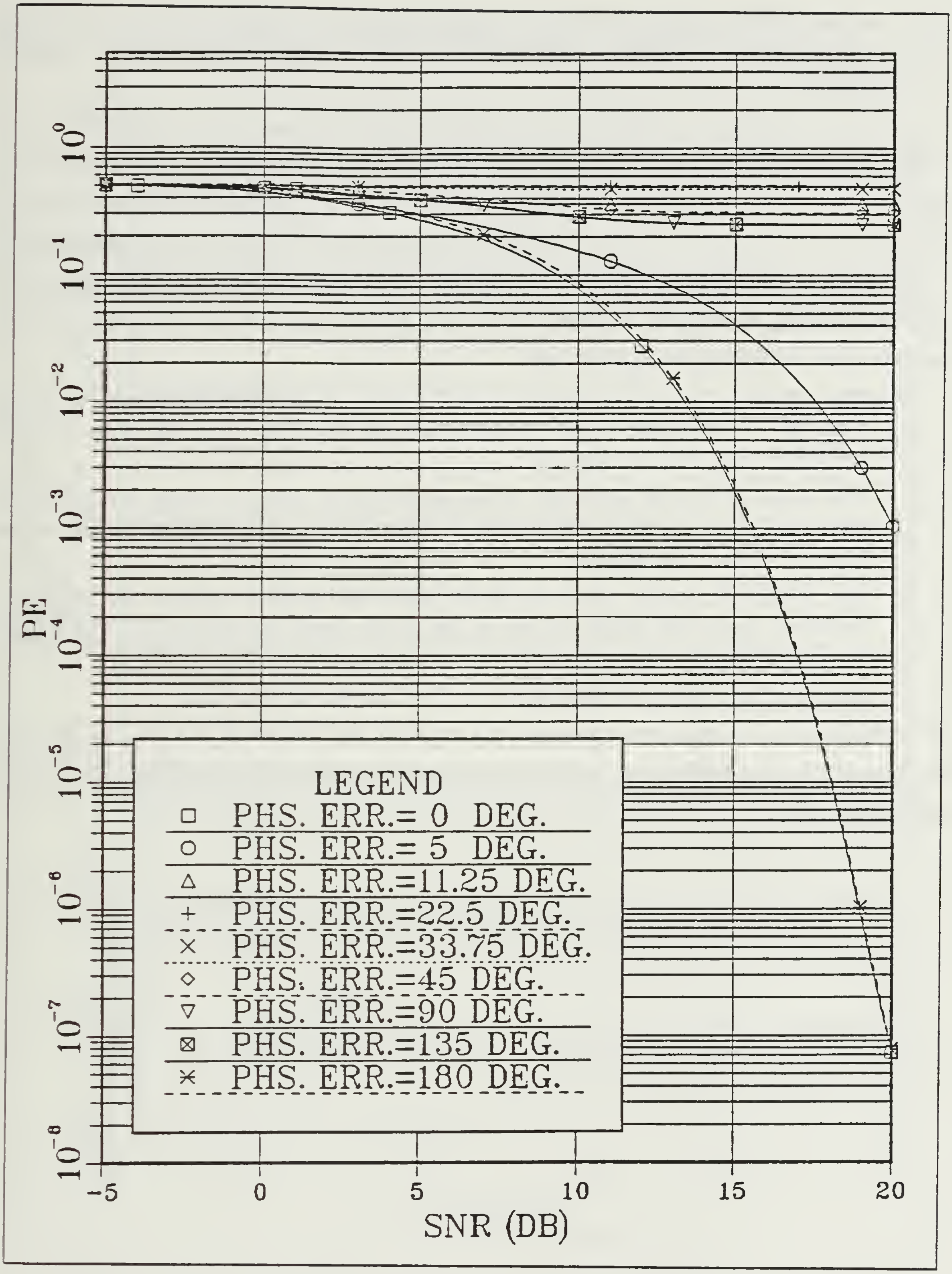

Figure 5.5 PE Versus SNR (dB) for 16-DPSK for Various Values of ε (α = 11.25°).

C. NUMERICAL RESULTS AND DISCUSSION OF RESULTS FOR 16-PSK AND 16-DPSK DBDR'S WITH RECEIVER LOCAL OSCILLATOR PHASE ERROR

In this chapter 16-PSK and 16-DPSK DBDRs have been analyzed when operating in the presence of a receiver local oscillator phase error. The performance results which have been obtained for the 16-ary case, are similar to those obtain for the 8-ary case. Hence, instead of repeating the conclusions similar to these presented for the 8-ary case, conclusions that are unique to the 16-ary case will be presented in this part. Main differences between 8-ary and 16-ary cases which are observed via analyses and numerical evaluations are

1. The optimum angles in minimum BER sense, α and β between the symbols becomes 11.25° and 33.75° respectively for 16-PSK case, as well as for 16-DSPK case, with $\alpha + \beta = 45^{\circ}$
2. The usage of 16-PSK provides better performance than 16-DPSK for $\varepsilon \leq 33.5^{\circ}$, however the actual BER values are too high for use in a practical system when ε greater than a few degrees.
3. The 16-DPSK system performs better than the 16-PSK system for $\varepsilon > 33.5^{\circ}$. However again the actual values of PE under these circumstances is too high for a practical system.
4. The 16-DPSK system is insensitive to phase errors of π radians.

Performance comparisons between the 16-PSK and 16-DPSK DBDRs are plotted on Figure 5.6, and BERs are plotted on Figures 5.7 and 5.8 respectively for various values of α with $\varepsilon = 0^{\circ}$

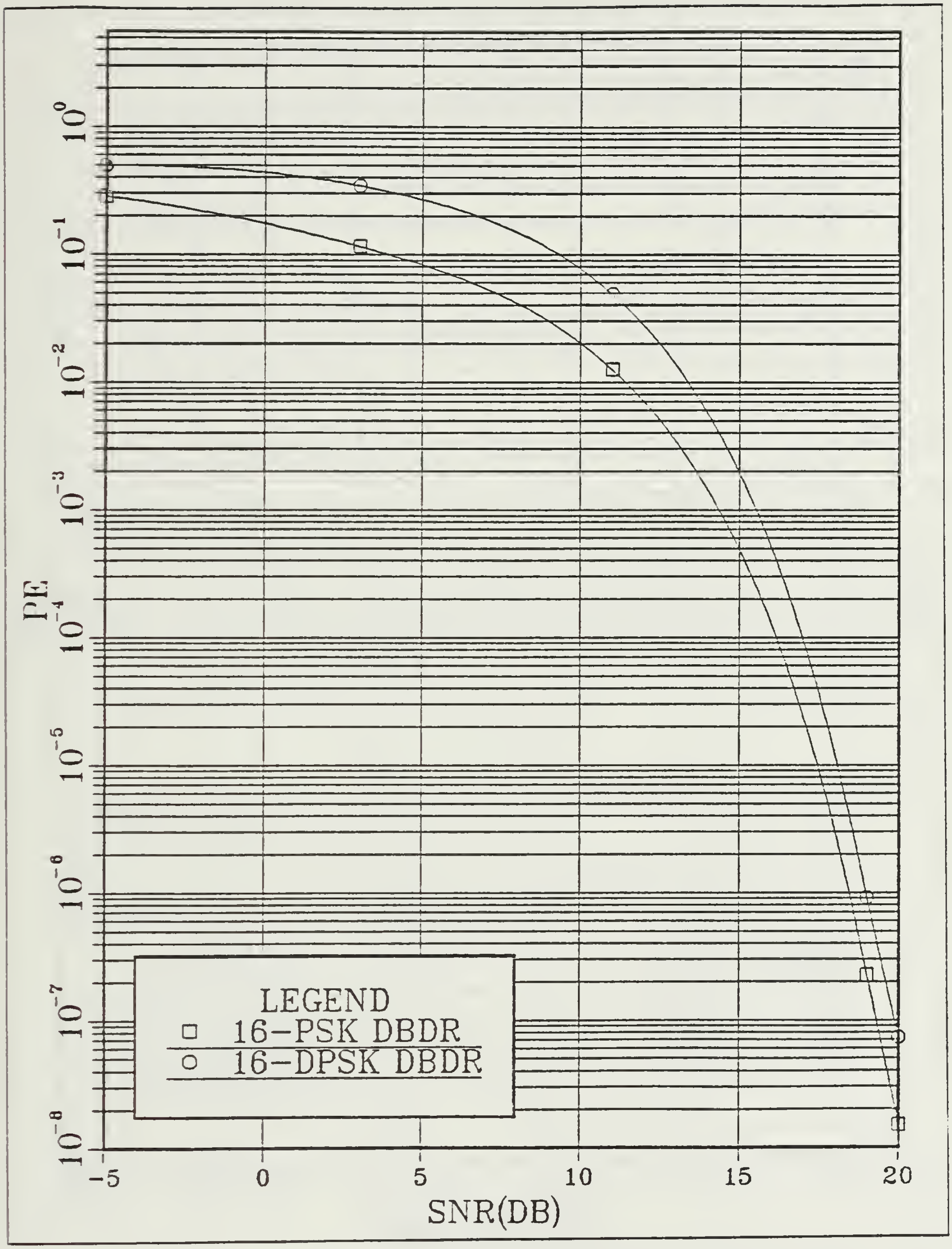

Figure 5.6 PE Versus SNR Comparison Plot for 16-PSK and 16-DPSK DBDR ($\alpha = 11.25°$, $\varepsilon = 0°$).

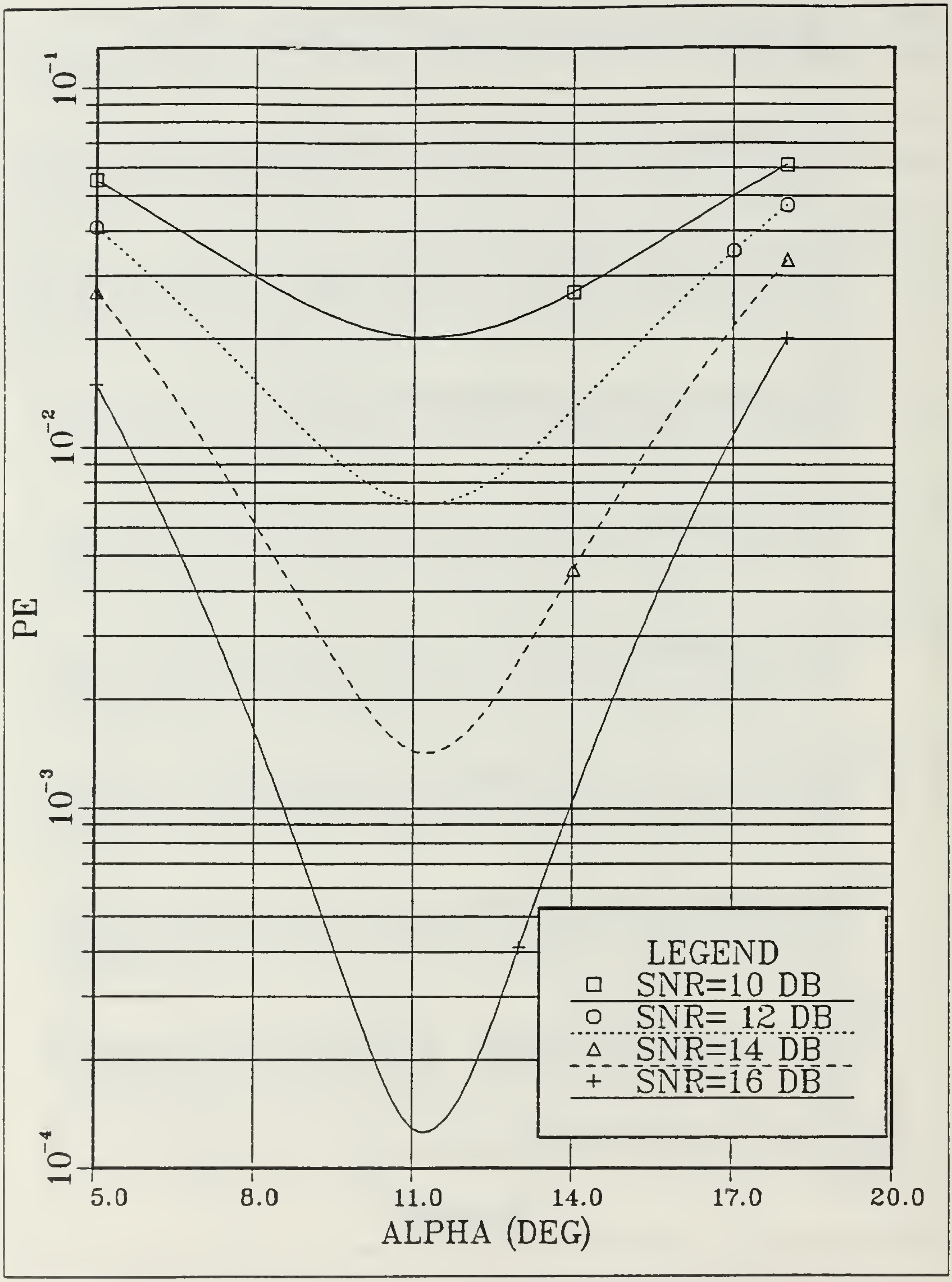

Figure 5.7 PE Versus α for 16-PSK DBDR for Various Values of SNR ($\varepsilon = 0°$).

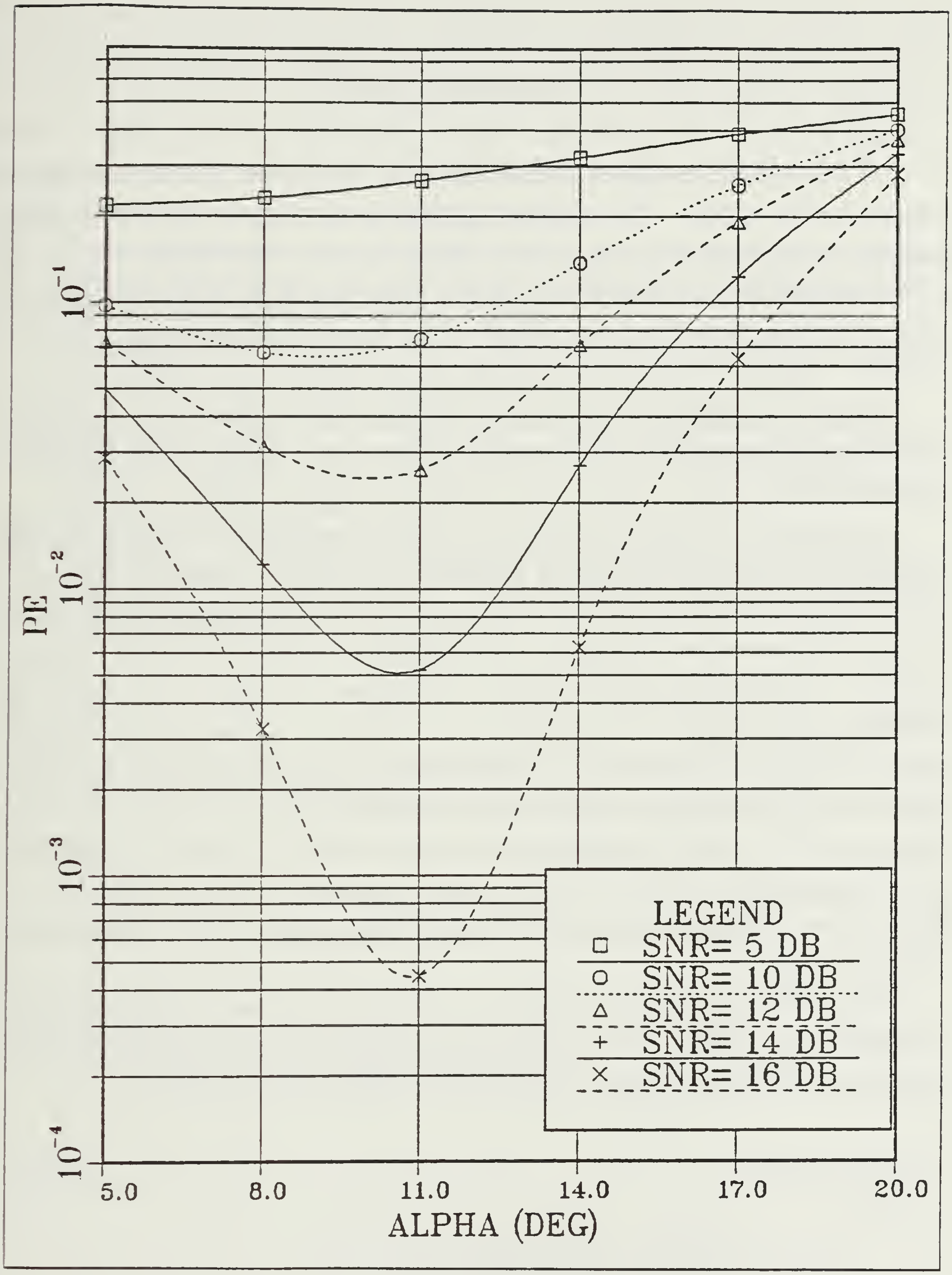

Figure 5.8 PE Versus α for 16-DPSK DBDR for Various Values of SNR (ε= 0°).

VI. CONCLUSIONS

In this research the use of DBDRs had been analyzed for M-PSK and MDPSK communication systems. The main assumptions in the analyses are a Gray coding mapping for the symbols and an Additive White Gaussian Noise channel model. It is further assumed that the receiver has a known phase error in its local oscillator and its effect on receiver performance is analyzed for the the special cases in which $M=8$ and $M=16$.

In addition to the analytical results, graphical results are presented in the form of plots of variations in PE for various values of phase angle α, SNR, and local oscillator phase error ε.

Optimum conventional M-PSK and M-DPSK receivers are designed to detect symbols while DBDRs are designed to detect bits. However, local oscillator phase errors degrade the performance of M-PSK and M-DPSK conventional receivers. Part of the work undertaken was intended to investigate the effect of such phase errors on DBDR's. The variations of PE versus ε are plotted in Figure 6.1 for 8-PSK and 8-DPSK, and in Figure 6.2 for 16-PSK and 16-DPSK. If ε (receiver local oscillator phase error) can be constrained to a small interval, namely $0^\circ \leq \varepsilon \leq 30^\circ$, the M-PSK system provides better performance than the M-DPSK system. For values of ε outside this range, both systems have high error rates that make the receivers unsuitable for practical applications. However, for complete phase reversal, i.e., $\varepsilon = 180^\circ$, the M-DPSK system suffers no performance penalty whereas the M-PSK system can not properly operate.

The performance degradation due to a phase error has been analyzed also from a different point of view. That is, when $\varepsilon \neq 0^\circ$, there is an increase in PE for both the M-PSK and M-DPSK receivers. The ratio of the phase error is given by

$$PER = PE_0 / PE_1 \qquad (6.1)$$

where

$PE_0 =$ Probability of bit error @ $\varepsilon = 0^\circ$

$PE_1 =$ Probability of bit error @ $\varepsilon \neq 0^\circ$

PER has been analyzed under various conditions for M-PSK and M-DPSK. The results have been plotted in Figures 6.3and 6.5 for $M=8$, and in Figures 6.6-6.8 for $M=16$, as a function of SNR for different values of ε. The plots demonstrate significant differences in the performance degradation of the $M=8$ versus $M=16$ case. However, for high SNR values, these differences tend to vanish.

The analysis results lead to the following conclusions

1. By using DBDRs, the error probability of each bit in the symbol state can be calculated independently and more importantly these error probabilities can be varied by changing the angle between symbols.
2. For optimum BER, α should be set to 22.5° for 8-PSK and 8-DPSK systems, while 16-PSK and 16-DPSK systems require $\alpha = 11.25°$ and $\beta + \alpha = 45°$.
3. The solution for the phase ambiguity problem cannot be achieved by using M-DPSK DBDR's, unless the phase ambiguity is 180°. Therefore, if the perfect coherency conditions cannot be achieved, the M-PSK system should be used whenever DBDR's are used.
4. When $\varepsilon = 0°$, the DPSK system exhibits good performance. However, if the $\varepsilon = 0°$ condition can be guaranteed there is no need to use the differential encoding scheme.

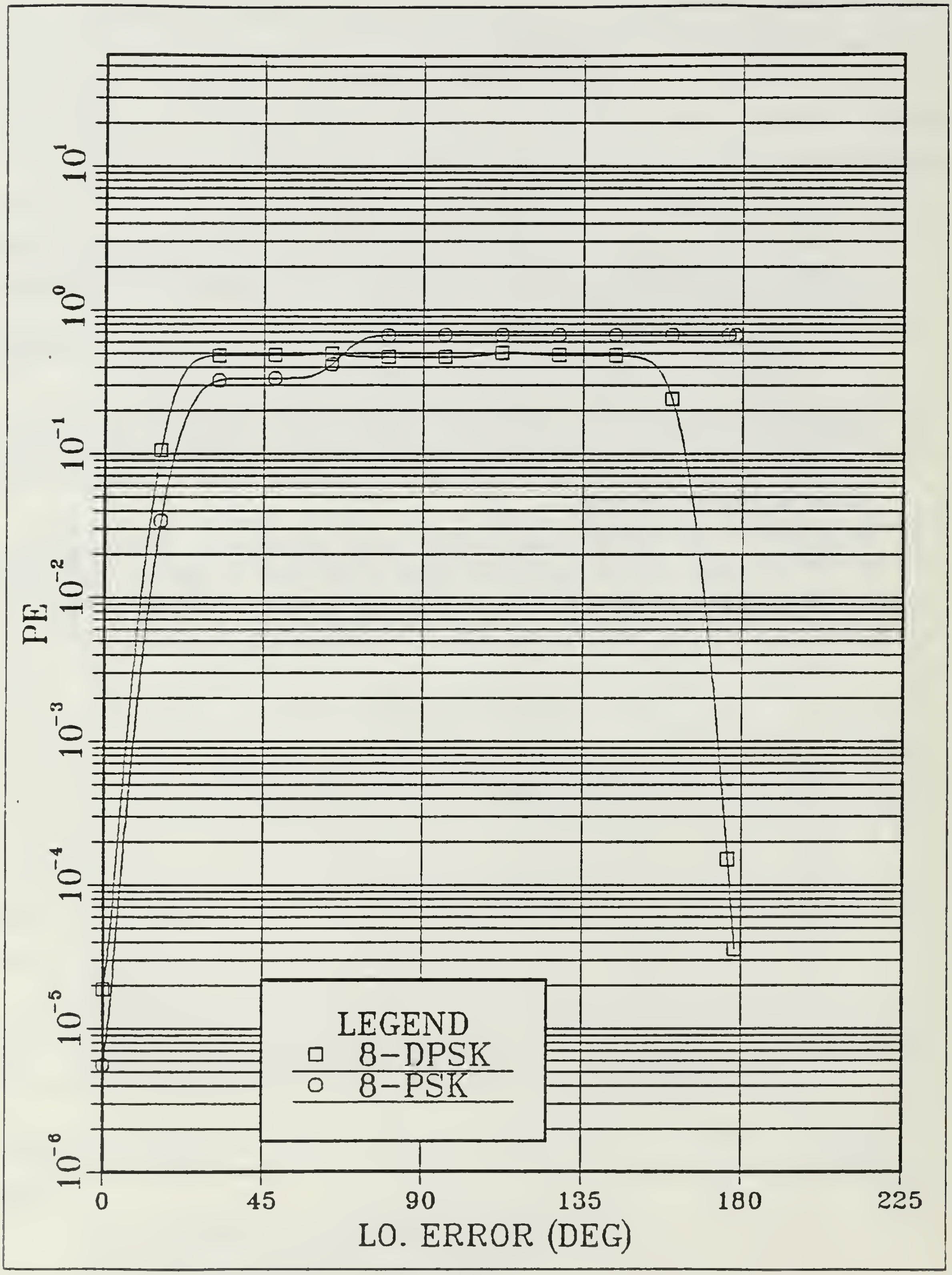

Figure 6.1 PE Versus ε for 8-PSK and 8-DPSK DBDR (SNR = 15 dB and α = 22.5°).

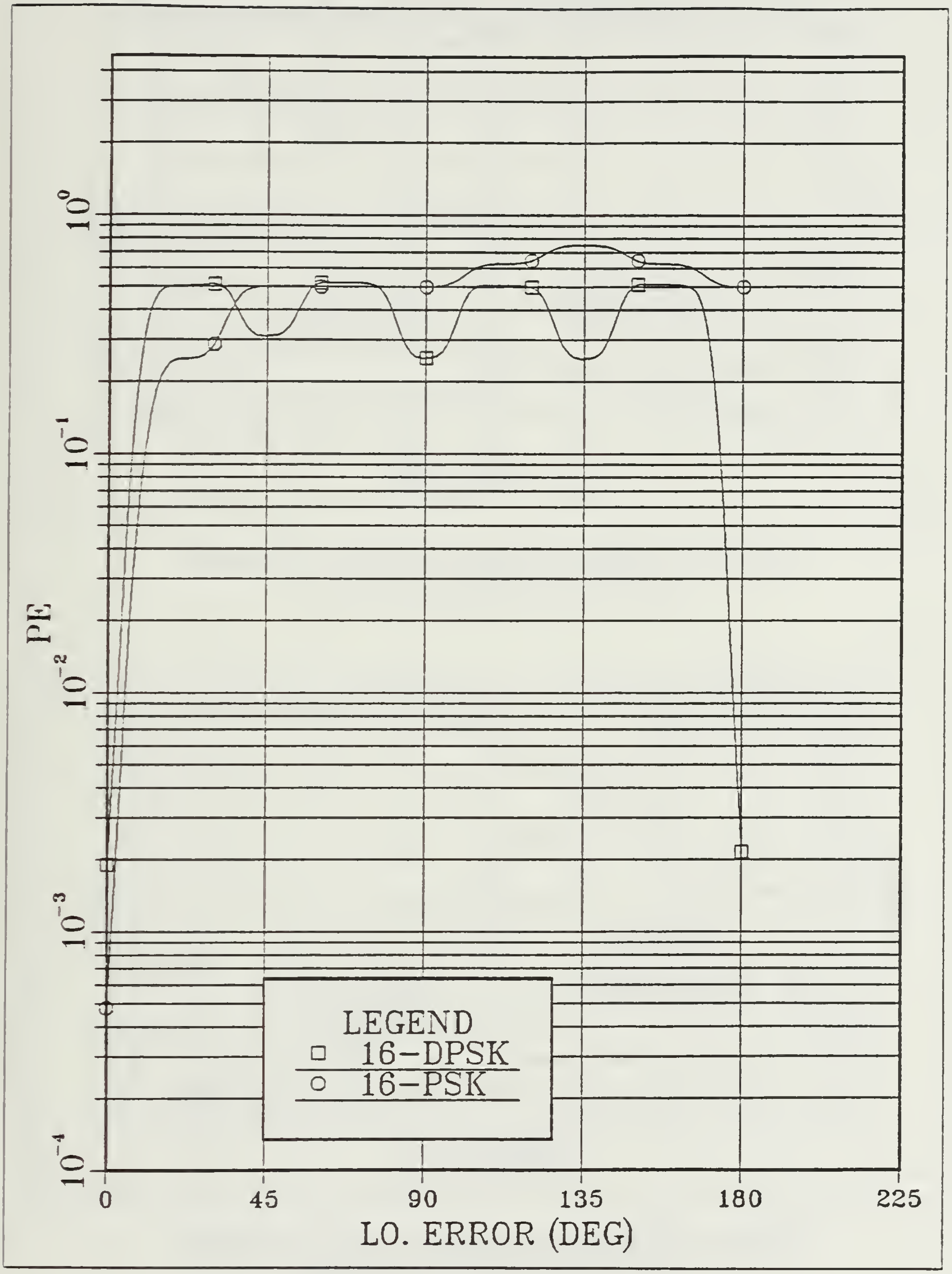

Figure 6.2 PE Versus ε for 16-PSK and 16-DPSK DBDR (SNR = 15 dB and α = 11.25°).

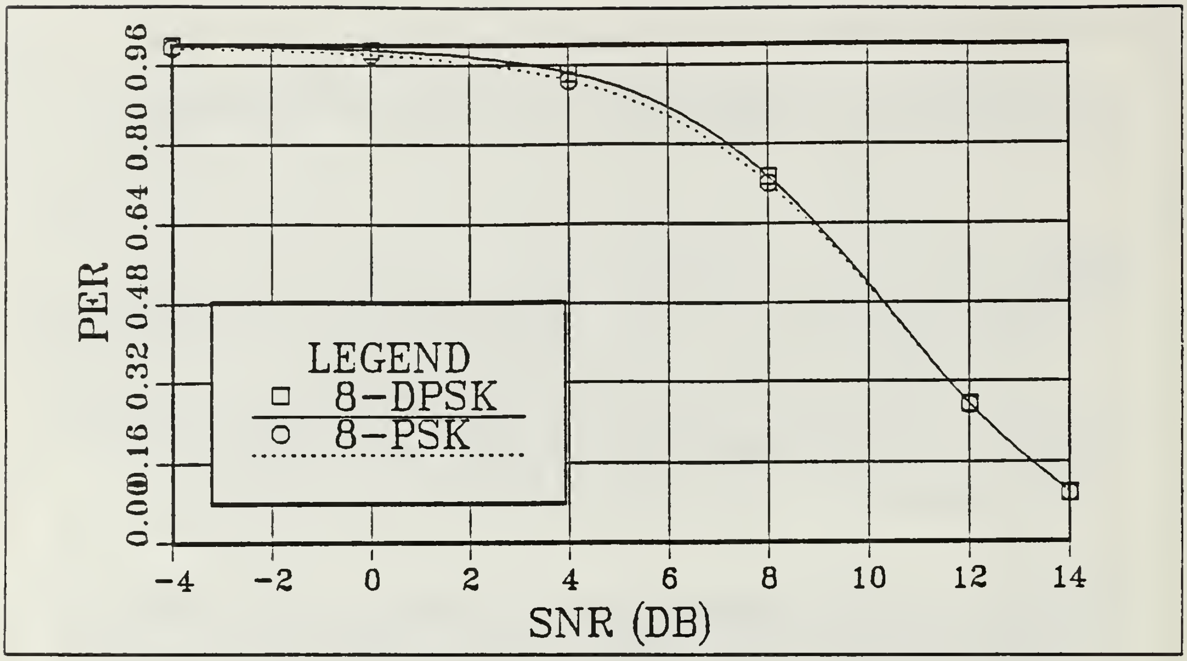

Figure 6.3 PER ($\varepsilon = 0°/\varepsilon = 5°$) Versus SNR for 8-PSK and 8-DPSK DBDR's ($\alpha = 22.5°$).

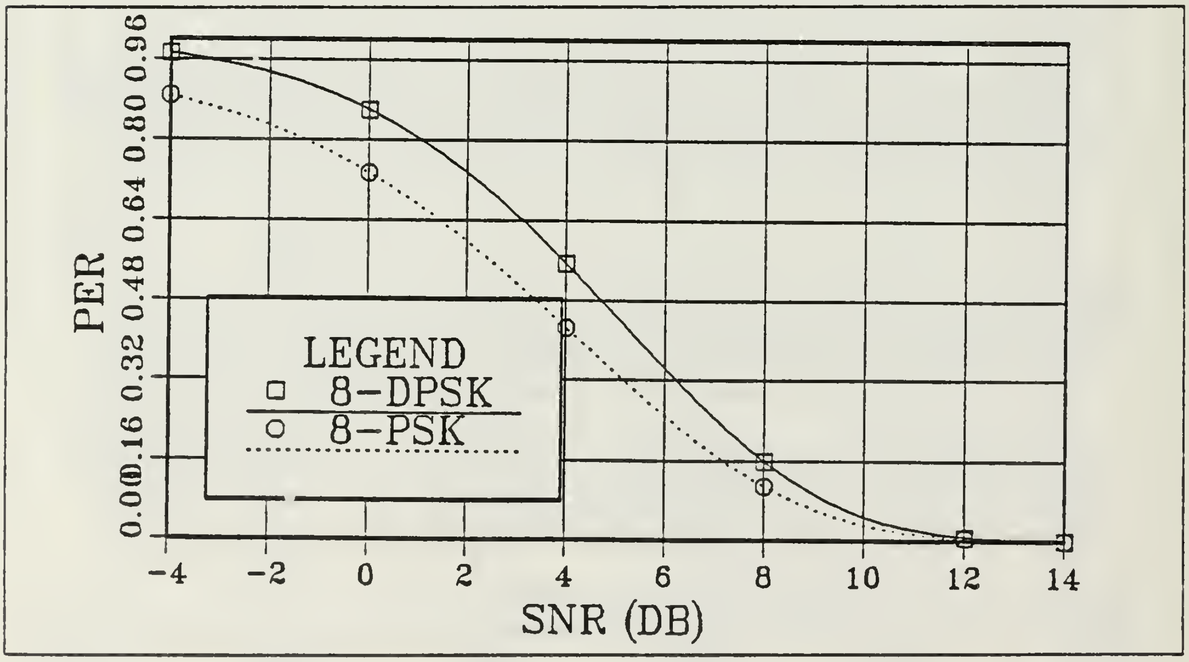

Figure 6.4 PER ($\varepsilon = 0°/\varepsilon = 22.5°$) Versus SNR for 8-PSK and 8-DPSK DBDR's ($\alpha = 22.5°$).

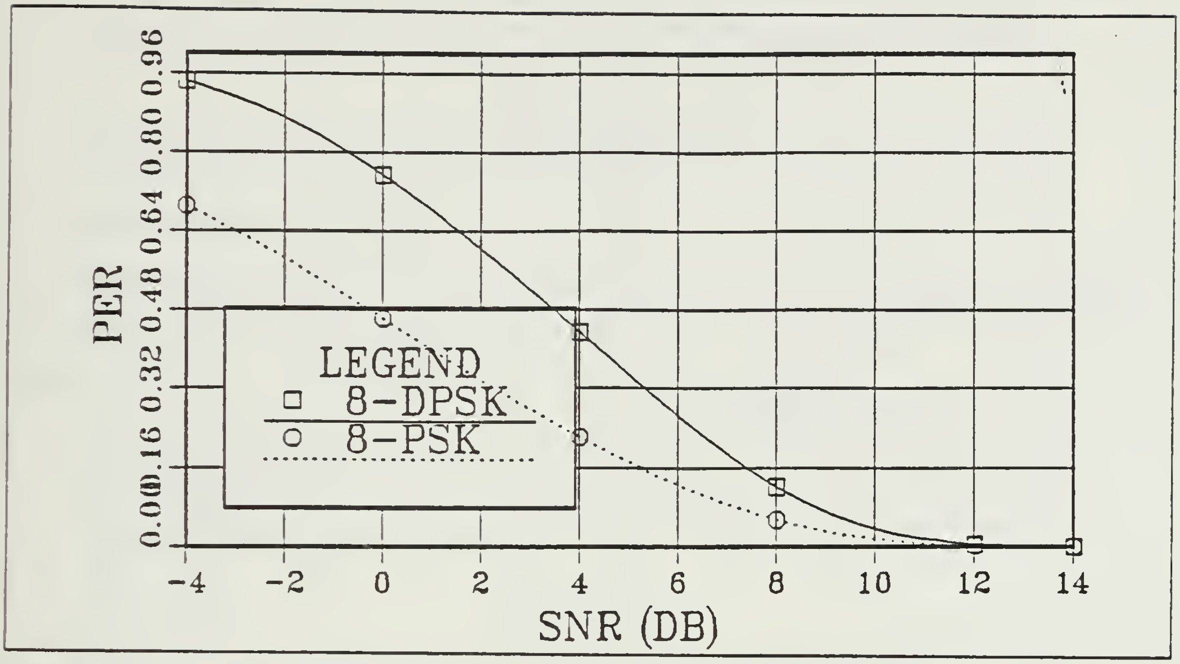

Figure 6.5 PER (ε = 0°/ε = 45°) Versus SNR for 8-PSK and 8-DPSK DBDR's (α = 22.5°).

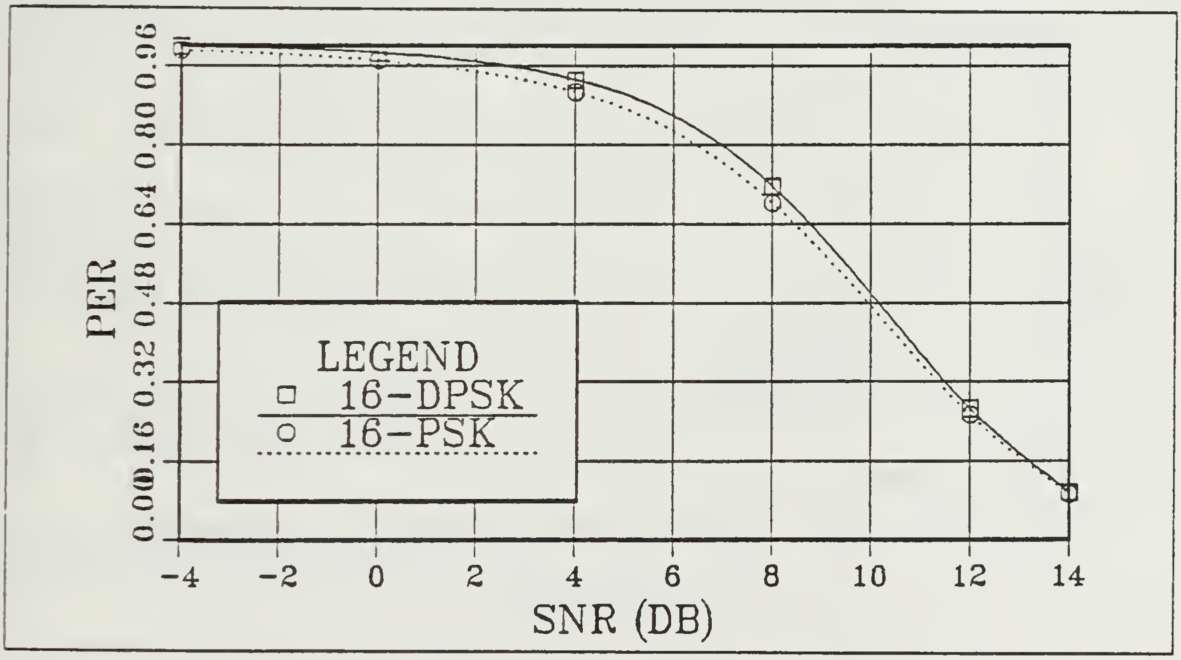

Figure 6.6 PER (ε = 0°/ε = 5°) Versus SNR for 16-PSK and 16-DPSK DBDR's (α = 11.25°).

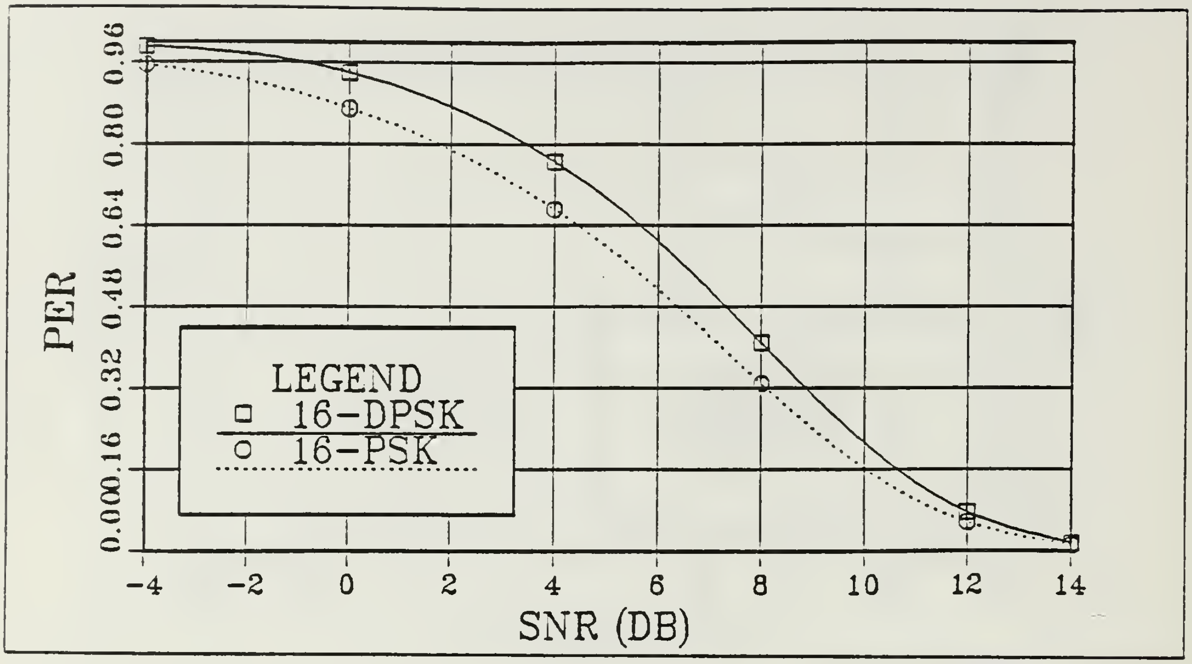

Figure 6.7 PER ($\varepsilon = 0°$, $\varepsilon = 11.25°$) Versus SNR for 16-PSK and 16-DPSK DBDR's ($\alpha =$ 11.25°).

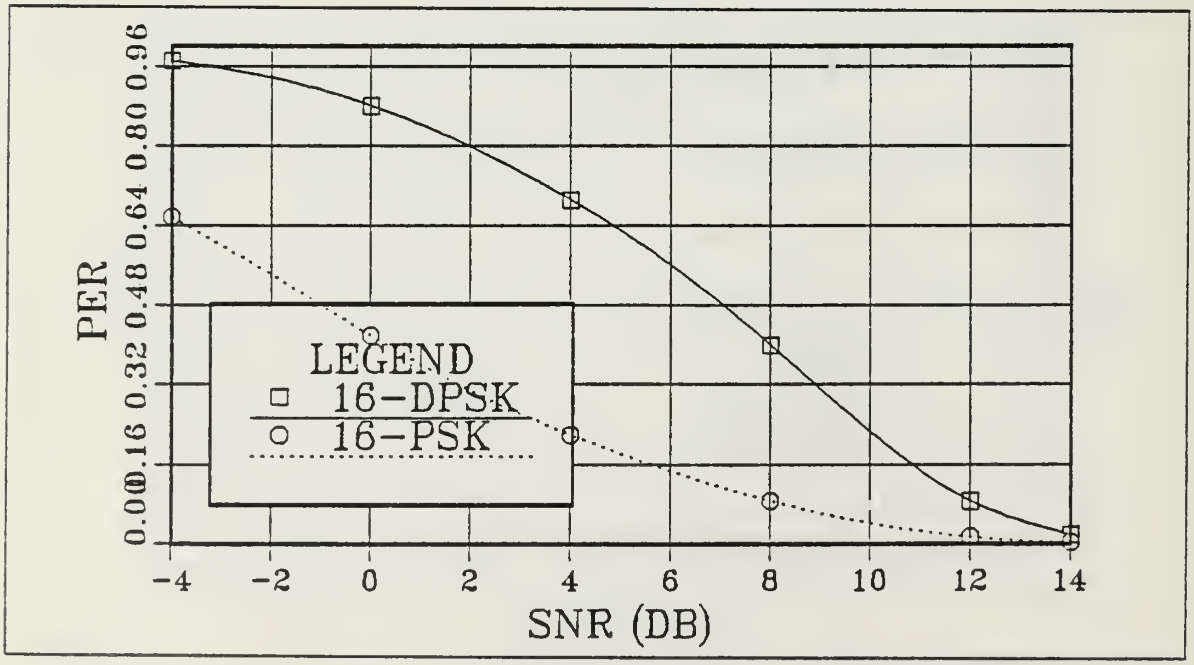

Figure 6.8 PER ($\varepsilon = 0°/\varepsilon = 45°$) Versus SNR for 16-PSK and 16-DPSK DBDR's ($\alpha =$ 11.25°).

APPENDIX A
DBDR INPUT/OUTPUT GENERATION PROGRAM (BELIZ)

1. USER GUIDE

This program generates all the possible input / output combinations in an M-DPSK communication system. The program runs interactively and sends outputs to the files

1. FILE FT08F001
2. FILE RCDATA
3. FILE XMDATA
4. FILE R
5. FILE ERR.

These files are used for information transfer between subroutines but are not required for program execution. It is possible to generate input / output combinations for 8-DPSK and 16-DPSK by using this program, while for 32-DPSK or 64-DPSK systems, dimensions and format statements must be changed accordingly, and a larger character variable set in the subroutine XMTR must be created. The program accepts only the integer equivalents of your data or symbol states. An example run of the program is shown below

```
run beliz
EXECUTION BEGINS...
 PLEASE SELECT ONE OF THEM
(1) IF YOU HAVE TRANSMITTER
(2) IF YOU HAVE RECEIVER

1
 PLEASE SELECT ONE OF THEM
(1) IF YOUR SYSTEM 8-DPSK
(2) IF YOUR SYSTEM 16-DPSK

1
ENTER THE INTEGER EQUIVALENTS OF SYMBOL STATES
IN THE ORDER OF COUNTER CLOCK WISE
           ENTER THE STATE #  1

7
           ENTER THE STATE #  2

5
           ENTER THE STATE #  3

1
           ENTER THE STATE #  4
```

```
3
          ENTER THE STATE #  5

2
          ENTER THE STATE #  6

0
          ENTER THE STATE #  7

4
          ENTER THE STATE #  8

6
ENTER THE INTEGER EQUIVALENTS OF THE
PHASE ANGLES OF YOUR CODING SYSTEM
            1*ALPHA= ?

0
            2*ALPHA= ?

1
            3*ALPHA= ?
3
            4*ALPHA= ?

2
            5*ALPHA= ?

6
            6*APLHA= ?

7
            7*ALPHA= ?

5
            8*ALPHA= ?

4
PLEASE ENTER THE SELECTION CORRECTLY
   ENTER 1 IF YOU NEED BINARY FORM OF SYMBOL AND
DATA STATES INCLUDING SYMBOL OUTPUTS
   ENTER 2 IF YOU NEED DECIMAL FORM OF SYMBOL AND
DATA STATES INCLUDING SYMBOL OUTPUTS
  ENTER 3 IF YOU WANT TO TURN BACK MAIN PROGRAM.

1
PLEASE ENTER THE SELECTION CORRECTLY
   ENTER 1 IF YOU NEED BINARY FORM OF SYMBOL AND
DATA STATES INCLUDING SYMBOL OUTPUTS
   ENTER 2 IF YOU NEED DECIMAL FORM OF SYMBOL AND
DATA STATES INCLUDING SYMBOL OUTPUTS
  ENTER 3 IF YOU WANT TO TURN BACK MAIN PROGRAM.

3
         ANALYSES RESULTS WAS SENT TO THE FILE FT08F001
```

2. SOURCE CODE

```
C
C          THIS PROGRAM  SIMULATES A  M-DPSK COMMUNICATION SYSTEM
C     PROGRAM RUNS TOTATLY INTERACTIVE THEN SENDS TO OUTPUTS FILES;
C     FILE FT08F001,FILE RCDATA,FILE XMDATA,FILE R,FILE ERR.
C          THESE FILES ARE USED FOR INFORMATION TRANSFER BETWEEN
C     SUBROUTINES BUT ARE NOT REQUIRED FOR PROGRAM EXECUTION.
C          YOU CAN SIMULATE 8-DPSK AND 16-DPSK BY USING THIS PROGRAM
C     FOR 32-DPSK OR 64-DPSK SYSTEMS YOU HAVE TO CHANGE DIMENSIONS
C     AND FORMAT STATEMENTS ACCORDINGLY,ALSO YOU HAVE TO EXPEND
C     CHARACTER VARIABLE SET IN THE SUBROUTINE XMTR.
C
C          OPERATION SPECIFICATIONS.......;
C
C          THE SYMBOL STATES REQUESTEd DEPEND ON YOUR SYSTEM AND CAN
C     BE ANY SEQUENCE.  MAIN THING IS; YOU HAVE TO SELECT YOUR REFERENCE
C     THEN ENTER THEM BY ONE STEP INCREMENTS,ALSO SAME SPECIFICATION
C     EXISTS FOR YOUR ANGLES.PROGRAM ONLY EXCEPTS THE INTEGER EQUIVALENTS
C     OF YOUR STATE OR ANGLE CODES.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C      VARIABLE DECLERATION
C
       INTEGER N,L,ORDER,TM(64,64),PS(64),TRDATA(256),M,KK,K,THETA(64)
       INTEGER MAT(64),KL,RMAT(64,64),TR(64),TH(64,64),RCVMAT(64,64),NN
       INTEGER QQU,QU,SQU,LD,IS,NT(1,8),KM(1,8),KN(1,8)
       INTEGER RCV(64,64),T(256),TD(256),NEXTST(256),PRESTA(256),FL
       INTEGER A,B,Y,KD,KY(1,8),KH(1,8),KJ(1,8),KB(1,8)
       CHARACTER*1 DATA(64,64)
       CHARACTER*16 SET
C
C      ***    CALLING SUBROUITINES     ***
       CALL INFO (ORDER,PS,THETA,NN,N,FL)
       IF (FL.EQ.1)THEN
       CALL XMTR (ORDER,FL)
       CALL XMSTA (THETA,TM,PS,ORDER)
       CALL XMDAT(TM,ORDER,THETA,NEXTST,PS,T,TD)
       CALL CONVER (N,NEXTST,NN,ORDER,FL)
       WRITE(6,1)
    1 FORMAT(T9,' ANALYSES RESULTS WAS SENT TO THE FILE FT08F001')
       ELSE
       CALL RCVR(ORDER)
       CALL RCVSTA (PS,ORDER,THETA,RCVMAT)
       CALL RCVDAT(TR,PS,ORDER,NEXTST,RCVMAT)
       CALL CONVER (N,NEXTST,NN,ORDER,FL)
       CALL ERROR(FL,THETA,RCVMAT,PS,ORDER,TR,TM,T,NEXTST,TD)
       WRITE(6,2)
    2 FORMAT(T9,' ANALYSES RESULTS WAS SENT TO THE FILE FT08F001',/,
      *T7,'ERROR ANALYSES RESULTS WAS SENT TO THE FILE ERR')
       ENDIF
       STOP
       END
C
C
       SUBROUTINE INFO(ORDER,PS,THETA,NN,N,FL)
C
C    THIS SUBROUTINE PROVIDES INFORMATIONS ABOUT OUR SYSTEM
C
       INTEGER N,L,ORDER,TM(64,64),PS(64),THETA(64),FL
       CHARACTER*1 DATA(64,64)
       WRITE (6,*) ' PLEASE SELECT ONE OF THEM'
       WRITE (6,*) '(1) IF YOU HAVE TRANSMITTER'
       WRITE (6,*) '(2) IF YOU HAVE RECEIVER'
       READ (6,*) FL
       WRITE (6,*) ' PLEASE SELECT ONE OF THEM'
       WRITE (6,*) '(1) IF YOUR SYSTEM 8-DPSK'
       WRITE (6,*) '(2) IF YOUR SYSTEM 16-DPSK'
       READ (6,*) L
       IF(L.EQ.1) THEN
       ORDER=8
       NN=63
       N=6
       ENDIF
       IF(L.EQ.2) THEN
       ORDER=16
       N=8
       NN=255
       ENDIF
       WRITE (6,*) 'ENTER THE INTEGER EQUIVALENTS OF SYMBOL STATES'
       WRITE (6,*) 'IN THE ORDER OF COUNTER CLOCK WISE'
       DO 11 K =1,ORDER
       WRITE(6,20)  K
  20  FORMAT(10X,'ENTER THE STATE #',I3)
       READ(6,*) PS(K)
  11  CONTINUE
       WRITE (6,*) 'ENTER THE INTEGER EQUIVALENTS OF THE '
       WRITE (6,*) 'PHASE ANGLES OF YOUR CODING SYSTEM '
       DO 33 L=1,ORDER
       WRITE (6,10) L
 10   FORMAT (10X,I3,'*ALPHA= ?')
       READ(6,*) THETA(L)
 33   CONTINUE
       RETURN
       END
C
       SUBROUTINE XMTR(ORDER,FL)
C
C
C        THIS SUBROUTINE PROVIDES GENERAL TRANSMITER MATRIX FOR
C     M-DPSK SYSTEM.
C        ITS OUTPUT CAN BE SEEN ON EITHER FILE FT08F001 OR FILE
C     XMDATA.
       INTEGER N,L,ORDER,FL
       CHARACTER*16 SET
       SET(1:16)= 'ABCDEFGHIJKLMNOP'
       OPEN(UNIT=11,FILE='XMDATA',STATUS='OLD')
       IF (FL.EQ.1) THEN
       WRITE (8,81)ORDER
 81   FORMAT(///,T15,'TRANSMITTER'S ENCODER MATRIX ANALYSIS
      * FOR',I3,'-DPSK'
      *,/,60('-'),//,T5,'GENERAL TRANSMITTER MATRIX',/,T2,31('-'))
       ELSE
       WRITE (8,88)
 88   FORMAT(///,T15,'TRANSMITTER INFORMATION FOR THE GIVEN RECEIVER'
      *,/,72('-'),//,T5,'GENERAL TRANSMITTER MATRIX',/,T2,31('-'))
       ENDIF
       DO 3 N=1,ORDER
       IF (N.EQ.1) THEN
       WRITE (8,*) SET(N:ORDER)
       WRITE (11,*) SET(N:ORDER)
       ELSE
       WRITE (8,*) SET(N:ORDER),SET(1:N-1)
       WRITE (11,*) SET(N:ORDER),SET(1:N-1)
       ENDIF
  3   CONTINUE
       CLOSE (11)
       RETURN
       END
C
        SUBROUTINE XMSTA(THETA,TM,PS,ORDER)
C
C     THIS SUBROUTINE CONVERTS GENERAL XMTR MATRIX IN TO OUR SPECIAL
C   XMTR SYMBOL MATRIX.
C     GENERAL XMTR MATRIX IS READ FROM FILE XMDATA.
C
       INTEGER TM(64,64),PS(64),THETA(64),ORDER
       CHARACTER*1 DATA(64,64)
       OPEN(UNIT=11,FILE='XMDATA',STATUS ='OLD')
       IF (ORDER.EQ.8)THEN
       WRITE(8,99)(THETA(I),I=1,ORDER)
 99   FORMAT(///,T5,'TRANSMITTER MATRIX ',/,31('-'),//,
      *'SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)'
      *,//,4X,8(I3),/,35('-'))
       READ (11,10) ((DATA(L,N),N=1,8),L=1,8)
 10   FORMAT (1X,8A1)
       ELSE
       WRITE(8,43)(THETA(I),I=1,ORDER)
 43   FORMAT(///,T5,'TRANSMITTER MATRIX ',/,31('-'),//,
      *'SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)'
      *,//,4X,16(I3),/,70('-'))
       READ (11,20) ((DATA(L,N),N=1,16),L=1,16)
 20   FORMAT (1X,16A1)
       ENDIF
       DO 3 K=1,ORDER
       DO 4 N=1,ORDER
       DO 6 M= 0,ORDER
       IF(ICHAR(DATA(K,N)).EQ.ICHAR(DATA(1,ORDER-M)))THEN
       TM(K,N)=PS(ORDER-M)
       ENDIF
  6   CONTINUE
  4   CONTINUE
       IF (ORDER.EQ.8)THEN
       WRITE(11,11) (TM(K,N),N=1,ORDER)
       WRITE(8,83) PS(K),(TM(K,N),N=1,ORDER)
 83   FORMAT(I3,'|',8(I3))
 11   FORMAT (8I2)
       ELSE
 77   WRITE(11,12) (TM(K,N),N=1,ORDER)
       WRITE(8,93) PS(K),(TM(K,N),N=1,ORDER)
 93   FORMAT(I3,'|',16(I3))
 12   FORMAT (16I3)
       ENDIF
  3   CONTINUE
       RETURN
       END
C
         SUBROUTINE XMDAT(TM,ORDER,THETA,NEXTST,PS,T,TD)
C
C          THIS SUBROUTINE FINDS THE NEXT SYMBOL STATE/S BY DETERMINING
C     XMTR MATRIX CONSTRUCTED BY SUB. XMSTA
C
C
       INTEGER TM(64,64),PS(64),TRDATA(256),KK,K,THETA(64),NN
       INTEGER ORDER,T(256),TD(256),NEXTST(256),PRESTA(256)
       DO 5 I=1,ORDER
       T(PS(I))=I
       TD(THETA(I))=I
  5   CONTINUE
       KK=0
       DO 7 K=0,ORDER-1
       DO 8 M=0,ORDER-1
       KK=KK+1
       NEXTST(KK)=TM(T(K),TD(M))
  8   CONTINUE
  7   CONTINUE
       RETURN
       END
C
        SUBROUTINE RCVR (ORDER)
C
C          THIS SUBROUTINE PREPARES GENERAL RECEIVER MATRIX AND
C     SENDS OUTPUTS TO THE "FILE RCDATA".
C
       INTEGER  ORDER,MAT(64),KL,RMAT(64,64)
       OPEN(UNIT=2,FILE='RCDATA',STATUS='OLD')
       WRITE (8,81)ORDER
 81   FORMAT(///,T15,'RECEIVER'S DECODER MATRIX ANALYSIS
      * FOR',I3,'-DPSK',/,
      *60('-'),//,T5,'GENERAL RECEIVER MATRIX',/,T2,31('-'))
       DO 3 I=0,ORDER-1
       MAT(I+1)=I
  3   CONTINUE
       DO 44 K=1,ORDER
       RMAT(1,K)=MAT(K)
 44   CONTINUE
       DO 55 L=2,ORDER
       DO 66 M=1,L-1
       RMAT(L,M)=MAT(ORDER-L+M+1)
 66   CONTINUE
       DO 77 N=L,ORDER
       RMAT(L,N)=MAT(N-L+1)
 77   CONTINUE
 55   CONTINUE
       DO 88 NK=1,ORDER
       IF (ORDER.EQ.8)THEN
       WRITE (2,10)(RMAT(NK,KL),KL=1,ORDER)
       WRITE (8,15)(RMAT(NK,KL),KL=1,ORDER)
  10  FORMAT(8(I2))
  15  FORMAT(8X,8(I2))
       ELSE
       WRITE (2,11)(RMAT(NK,KL),KL=1,ORDER)
       WRITE (8,16)(RMAT(NK,KL),KL=1,ORDER)
  11  FORMAT(16(I2))
  16  FORMAT(8X,16(I2))
       ENDIF
 88   CONTINUE
       CLOSE(2)
       RETURN
       END
C
       SUBROUTINE  RCVSTA (PS,ORDER,THETA,RCVMAT)
C
C        THIS SUBROUTINE PREPARES SPECIFIC RECEIVER MATRIX UNDER
C     DEFINED STATES.IT READS GENERAL RECEIVER MATRIX FORM FILE
C     RCDATA WHICH HAD BEEN CREATED BY THE SUBROUTINE RCVDAT.
C       OUTPUT ARE SENT TO THE FILE R.
C
       INTEGER TH(64,64),ORDER,PS(64),THETA(64),RCVMAT(64,64)
       OPEN(UNIT=2,FILE='RCDATA',STATUS='OLD')
       OPEN(UNIT=20,FILE='R',STATUS='OLD')
       IF (ORDER.EQ.8)THEN
       WRITE(8,99)(PS(L),L=1,ORDER),(THETA(I),I=1,ORDER)
 99   FORMAT(///,T5,'RECEIVER MATRIX IS',/,31('-'),//,
      *'SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)',//,4X,8(I3))
       ELSE
       WRITE(8,199)(PS(L),L=1,ORDER)
199   FORMAT(///,T5,'RECEIVER MATRIX IS',/,31('-'),//,
      *'SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)'
      *,//,4X,16(I3))
       ENDIF
       DO 1 I=1,ORDER
       IF (ORDER.EQ.8)THEN
       READ(2,11) (TH(I,N),N=1,ORDER)
 11   FORMAT(8I2)
       ELSE
       READ(2,12) (TH(I,N),N=1,ORDER)
 12   FORMAT(16I2)
       ENDIF
       DO 2 K=1,ORDER
       RCVMAT(I,K)=THETA(TH(I,K)+1)
   2  CONTINUE
       IF (ORDER.EQ.8)THEN
       WRITE(20,13) (RCVMAT(I,N),N=1,ORDER)
 13   FORMAT(8I2)
       WRITE(8,83) PS(I),(RCVMAT(I,N),N=1,ORDER)
 83   FORMAT(I3,'|',8(I3))
```

```
       ELSE
       WRITE(8,93) PS(I),(RCVMAT(I,N),N=1,ORDER)
 93   FORMAT(I3,'|',16(I3))
       WRITE(20,14) (RCVMAT(I,N),N=1,ORDER)
 14   FORMAT(16I2)
       ENDIF
  1   CONTINUE
       CLOSE(20)
       RETURN
       END
C
        SUBROUTINE RCVDAT(TR,PS,ORDER,NEXTST,RCVMAT)
C
C         THIS SUBROUTINE READS SPECIFIC RECEIVER MATRIX WHICH  HAD
C       BEEN CREATED BY SUBROUTINE RCVSTA,THEN SEARCHS FOR THE RCVR
C       OUTPUT.
C
C
       INTEGER RCVMAT(64,64),PS(64),KK,ORDER
       INTEGER TR(64),NEXTST(256)
       OPEN(UNIT=20,FILE='R',STATUS='OLD')
       DO 23 LL=1,ORDER
       IF (ORDER.EQ.8)THEN
       READ (20,24)(RCVMAT(LL,KM),KM=1,ORDER)
 24   FORMAT(8(I2))
       ELSE
       READ (20,94)(RCVMAT(LL,KM),KM=1,ORDER)
 94   FORMAT(16(I2))
       ENDIF
 23   CONTINUE
       DO 2 I=1,ORDER
       M=PS(I)
       TR(M)=I
  2   CONTINUE
       KK=0
       DO 3 M=0,ORDER-1
       DO 4 K=0,ORDER-1
       KK=KK+1
       NEXTST(KK)=RCVMAT(TR(M),TR(K))
  4   CONTINUE
  3   CONTINUE
       RETURN
       END
C
C
       SUBROUTINE CONVER(N,NEXTST,NN,ORDER,FL)
C
C
       INTEGER  N,S,QU,SU,LLL,LLK(50),MMM,NN,FL,MM,M,K(9,1024)
       INTEGER  MW,NT(256,4),KZ,ORDER,NEXTST(256),ST,QQU,KD
C
 300  WRITE (6,*)  'PLEASE ENTER THE SELECTION CORRECTLY'
       WRITE (6,*)  '    ENTER 1 IF YOU NEED BINARY FORM OF SYMBOL AND '
       WRITE (6,*)  'DATA STATES INCLUDING SYMBOL OUTPUTS'
       WRITE (6,*)  '    ENTER 2 IF YOU NEED DECIMAL FORM OF SYMBOL AND '
       WRITE (6,*)  'DATA STATES INCLUDING SYMBOL OUTPUTS'
       WRITE (6,*)  '   ENTER 3 IF YOU WANT TO TURN BACK MAIN PROGRAM.'
       READ (6,*) S
       IF (S.EQ.1) GOTO 100
       IF (S.EQ.2) GOTO 200
       IF (S.EQ.3) GOTO 400
       IF (S.NE.1.OR.S.NE.2.OR.S.NE.3) GOTO 300
 200  IF (FL.EQ.1) THEN
       WRITE(8,6)
    6 FORMAT (//,8X,'DATA STATE',5X,'SYMBOL STATE',5X,'SYMBOL')
       ELSE
       WRITE(8,21)
  21  FORMAT (//,8X,'DATA STATE',5X,'SYMBOL STATE',5X,'SYMBOL')
       ENDIF
       KK=0
       DO 7 KZ=0,ORDER-1
       DO 8 M=0,ORDER-1
       KK=KK+1
       WRITE(8,9)  KZ,M, NEXTST(KK)
   9  FORMAT (10X,I3,15X,I3,15X,I3)
   8  CONTINUE
   7  CONTINUE
       GOTO 300
 100  IF (FL.EQ.1) THEN
       WRITE(8,47)
  47  FORMAT (//,7X,'DATA STATE',10X,'SYMBOL STATE',10X,
     *'SYMBOL',/,72('-'))
       ELSE
       WRITE(8,77)
  77  FORMAT (//,7X,'SYMBOL STATE',10X,'DATA STATE',10X,
     *'SYMBOL',/,72('-'))
       ENDIF
       IF(ORDER.EQ.8)THEN
       KD=3
       ELSE
       KD=4
       ENDIF
       DO 44 L=1,NN+1
       QQU=NEXTST(L)
       DO 33 SR=KD,1,-1
       NT(L,SR)=MOD(QQU,2)
       QQU=QQU/2
  33  CONTINUE
  44  CONTINUE
       DO 23 MW=0,NN
       QU=MW
       DO 22 LLL=N,1,-1
       K(MW,LLL)=MOD(QU,2)
       QU=QU/2
 22   CONTINUE
       IF (ORDER.EQ.8)THEN
       WRITE(8,45) MW,(K(MW,LK),LK=1,N),(NT(MW+1,ST),ST=1,KD)
 45   FORMAT (I3,2X,3(2X,I2,2X),3X,3(2X,I2,2X),3X,3(2X,I2,2X))
       ELSE
       WRITE(8,55) MW,(K(MW,LK),LK=1,N),(NT(MW+1,ST),ST=1,KD)
 55   FORMAT (I3,2X,4(2X,I2,1X),3X,4(2X,I2,1X),3X,4(2X,I2,1X))
       ENDIF
       WRITE (8,46)
 46   FORMAT (72('-'))
 23   CONTINUE
       GOTO 300
 400  RETURN
       END
C
       SUBROUTINE ERROR(FL,THETA,RCVMAT,PS,ORDER,TR,TM,T,NEXTST,TD)
C
C      THIS SUBROUTINE LOCATES UNDETECTED ERROR COMBINATIONS.
C
C
       INTEGER QQU,QU,SQU,PS(64),ORDER,RCVMAT(64,64),N,TR(64),TM(64,64)
       INTEGER TD(256),T(256),A,B,Y,KD,KN(1,8),KJ(1,8),KB(1,8),KY(1,8)
       INTEGER FL,IS,NT(1,8),KM(1,8),NEXTST(256),THETA(64),KH(1,8)
       OPEN(UNIT=1,FILE='ERR',STATUS='OLD')
       OPEN(UNIT=15,FILE='ERDATA',STATUS='OLD')
       WRITE (6,*) '    IF YOU WANT TO HAVE UNDETECTABLE ERROR'
       WRITE(6,*) 'ANALYSES ENTER "1",OTHERWISE ENTER "2"'
       READ (6,*) LD
       IF (ORDER.EQ.8)THEN
       KD=3
       ELSE
       KD=4
       ENDIF
       IF(LD.EQ.1)THEN
       WRITE (6,*)'   WHAT KIND OF ANALYSES OUTPUT DO YOU WANT'
       WRITE (6,*)'  (1) IN BINARY FORM'
```

```
      WRITE (6,*)'  (2) IN DECIMAL FORM'
      READ (6,*) IS
      WRITE(1,13)
13   FORMAT(////,72('#'),//,7X,'CORRECT RECEPTION COMBINATIONS IN PRESE
     *NCE OF ERROR',//,72('#'))
      WRITE(1,12)
 12  FORMAT(//,T5,'INPUT',T17,'PREVIOUS',T30,'PRESENT',T43,'PREVIOUS',
     *T55,'PRESENT',T68,'OUTPUT',/,T15,'TRANSMISSION',T28,'TRANSMISSION'
     *,T42,'RECEPTION',T54,'RECEPTION',/,79('-'))
      ELSE
      GOTO 111
      ENDIF
      FL=2
      CALL XMTR (ORDER,FL)
      CALL XMSTA (THETA,TM,PS,ORDER)
      CALL XMDAT(TM,ORDER,THETA,NEXTST,PS,T,TD)
      DO 1 I=0,ORDER-1
      DO 2 K=0,ORDER-1
      A=TM(T(I),TD(K))
      B=RCVMAT(TR(I),TR(A))
      IF(IS.EQ.1)THEN
      QQU=A
      DO 333 JJ=KD,1,-1
      NT(1,JJ)=MOD(QQU,2)
      QQU=QQU/2
333  CONTINUE
      QU=K
      DO 222 LM=KD,1,-1
      KM(1,LM)=MOD(QU,2)
      QU=QU/2
222  CONTINUE
      SQU=I
      DO 444 LN=KD,1,-1
      KN(1,LN)=MOD(SQU,2)
      SQU=SQU/2
444  CONTINUE
      SQU=B
      DO 555 LN=KD,1,-1
      KB(1,LN)=MOD(SQU,2)
      SQU=SQU/2
555  CONTINUE
      IF (ORDER.EQ.8)THEN
      WRITE(1,45)(KM(1,L),L=1,KD),(KN(1,KK),KK=1,KD),(NT(1,IT),IT=1,KD)
     *,(KN(1,KK),KK=1,KD),(NT(1,IT),IT=1,KD),(KB(1,IJ),IJ=1,KD)
45   FORMAT(T3,3(I3),T15,3(I3),T28,3(I3),T42,3(I3),T54,3(I3),T65,3(I3))
      WRITE(15,85)(KN(1,KK),KK=1,KD),(NT(1,IT),IT=1,KD)
     *,(KB(1,IJ),IJ=1,KD)
85   FORMAT(9(I2))
      ELSE
      WRITE(1,55)(KM(1,L),L=1,KD),(KN(1,KK),KK=1,KD),(NT(1,IT),IT=1,KD)
     *,(KN(1,KK),KK=1,KD),(NT(1,IT),IT=1,KD),(KB(1,IJ),IJ=1,KD)
55   FORMAT(T1,4(I2),T15,4(I2),T28,4(I2),T42,4(I2),T54,4(I2),T65,4(I2))
      WRITE(15,95)(KN(1,KK),KK=1,KD),(NT(1,IT),IT=1,KD)
     *,(KB(1,IJ),IJ=1,KD)
95   FORMAT(12(I2))
      ENDIF
      ELSE
      WRITE(1,10) K,I,A,I,A,B
10   FORMAT(T10,I3,T25,I3,T35,I3,T48,I3,T58,I3,T71,I3)
      ENDIF
      DO 3 L=0,ORDER-1
      DO 4 J=0,ORDER-1
      IF(L.EQ.I.AND.J.EQ.A)THEN
      GOTO 100
      ELSE
      Y=RCVMAT(TR(L),TR(J))
      IF(Y.EQ.B)THEN
      IF(IS.EQ.1)THEN
      SQU=L
      DO 666 LN=KD,1,-1
      KH(1,LN)=MOD(SQU,2)
      SQU=SQU/2
 666 CONTINUE
      SQU=J
      DO 777 LN=KD,1,-1
      KJ(1,LN)=MOD(SQU,2)
      SQU=SQU/2
 777 CONTINUE
      SQU=Y
      DO 888 LN=KD,1,-1
      KY(1,LN)=MOD(SQU,2)
      SQU=SQU/2
 888 CONTINUE
      IF(ORDER.EQ.8)THEN
      WRITE(1,48)(KH(1,KK),KK=1,KD),(KJ(1,KK),KK=1,KD)
     *,(KY(1,IT),IT=1,KD)
 48  FORMAT (T42,3(I3),T54,3(I3),T65,3(I3))
      WRITE(15,999)(KH(1,KK),KK=1,KD),(KJ(1,KK),KK=1,KD)
     *,(KY(1,IT),IT=1,KD)
999  FORMAT (9(I2))
      ELSE
      WRITE(1,49)(KH(1,KK),KK=1,KD),(KJ(1,KK),KK=1,KD)
     *,(KY(1,IT),IT=1,KD)
 49  FORMAT (T42,4(I2),T54,4(I2),T65,4(I2))
      WRITE(15,949)(KH(1,KK),KK=1,KD),(KJ(1,KK),KK=1,KD)
     *,(KY(1,IT),IT=1,KD)
949  FORMAT (12(I2))
      ENDIF
      ELSE
      WRITE(1,20) L,J,Y
 20  FORMAT(T48,I3,T58,I3,T71,I3)
      ENDIF
      ENDIF
 100 ENDIF
 4   CONTINUE
 3   CONTINUE
      WRITE (1,46)
 46  FORMAT (79('-'))
 2   CONTINUE
 1   CONTINUE
 111 CLOSE(1)
      CLOSE(15)
      RETURN
      END
```

APPENDIX B
COMPUTER GENERATED ENCODER AND DECODER OUTPUTS

1. 8-DPSK TRANSMITTER

```
          TRANSMITTER'S ENCODER  MATRIX TABLE FOR  8-DPSK
------------------------------------------------------------------------

     GENERAL TRANSMITTER MATRIX
 --------------------------------
 ABCDEFGH
 BCDEFGHA
 CDEFGHAB
 DEFGHABC
 EFGHABCD
 FGHABCDE
 GHABCDEF
 HABCDEFG


     TRANSMITTER MATRIX
--------------------------------
SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)
/     0  1  3  2  6  7  5  4
------------------------------------
   7|  7  5  1  3  2  0  4  6
   5|  5  1  3  2  0  4  6  7
   1|  1  3  2  0  4  6  7  5
   3|  3  2  0  4  6  7  5  1
   2|  2  0  4  6  7  5  1  3
   0|  0  4  6  7  5  1  3  2
   4|  4  6  7  5  1  3  2  0
   6|  6  7  5  1  3  2  0  4

             SYMBOL STATE               DATA STATE                SYMBOL
---------------------------------------------------------------------------
  0     0     0     0        0     0     0        0     0     0
---------------------------------------------------------------------------
  1     0     0     0        0     0     1        1     0     0
---------------------------------------------------------------------------
  2     0     0     0        0     1     0        1     1     1
---------------------------------------------------------------------------
  3     0     0     0        0     1     1        1     1     0
---------------------------------------------------------------------------
  4     0     0     0        1     0     0        0     1     0
---------------------------------------------------------------------------
  5     0     0     0        1     0     1        0     1     1
---------------------------------------------------------------------------
  6     0     0     0        1     1     0        1     0     1
---------------------------------------------------------------------------
  7     0     0     0        1     1     1        0     0     1
---------------------------------------------------------------------------
  8     0     0     1        0     0     0        0     0     1
---------------------------------------------------------------------------
  9     0     0     1        0     0     1        0     1     1
---------------------------------------------------------------------------
 10     0     0     1        0     1     0        0     0     0
---------------------------------------------------------------------------
 11     0     0     1        0     1     1        0     1     0
---------------------------------------------------------------------------
 12     0     0     1        1     0     0        1     0     1
---------------------------------------------------------------------------
```

13	0	0	1	1	0	1	1	1	1
14	0	0	1	1	1	0	1	0	0
15	0	0	1	1	1	1	1	1	0
16	0	1	0	0	0	0	0	1	0
17	0	1	0	0	0	1	0	0	0
18	0	1	0	0	1	0	1	1	0
19	0	1	0	0	1	1	1	0	0
20	0	1	0	1	0	0	0	1	1
21	0	1	0	1	0	1	0	0	1
22	0	1	0	1	1	0	1	1	1
23	0	1	0	1	1	1	1	0	1
24	0	1	1	0	0	0	0	1	1
25	0	1	1	0	0	1	0	1	0
26	0	1	1	0	1	0	1	0	0
27	0	1	1	0	1	1	0	0	0
28	0	1	1	1	0	0	0	0	1
29	0	1	1	1	0	1	1	0	1
30	0	1	1	1	1	0	1	1	0
31	0	1	1	1	1	1	1	1	1
32	1	0	0	0	0	0	1	0	0
33	1	0	0	0	0	1	1	1	0
34	1	0	0	0	1	0	1	0	1
35	1	0	0	0	1	1	1	1	1
36	1	0	0	1	0	0	0	0	0
37	1	0	0	1	0	1	0	1	0
38	1	0	0	1	1	0	0	0	1
39	1	0	0	1	1	1	0	1	1
40	1	0	1	0	0	0	1	0	1
41	1	0	1	0	0	1	0	0	1
42	1	0	1	0	1	0	0	1	0
43	1	0	1	0	1	1	0	1	1
44	1	0	1	1	0	0	1	1	1
45	1	0	1	1	0	1	1	1	0
46	1	0	1	1	1	0	0	0	0
47	1	0	1	1	1	1	1	0	0

48	1	1	0	0	0	0	1	1	0
49	1	1	0	0	0	1	1	1	1
50	1	1	0	0	1	0	0	0	1
51	1	1	0	0	1	1	1	0	1
52	1	1	0	1	0	0	1	0	0
53	1	1	0	1	0	1	0	0	0
54	1	1	0	1	1	0	0	1	1
55	1	1	0	1	1	1	0	1	0
56	1	1	1	0	0	0	1	1	1
57	1	1	1	0	0	1	1	0	1
58	1	1	1	0	1	0	0	1	1
59	1	1	1	0	1	1	0	0	1
60	1	1	1	1	0	0	1	1	0
61	1	1	1	1	0	1	1	0	0
62	1	1	1	1	1	0	0	1	0
63	1	1	1	1	1	1	0	0	0

2. 8-DPSK RECEIVER

```
             RECEIVER'S DECODER MATRIX TABLE FOR  8-DPSK
-----------------------------------------------------------------
       GENERAL RECEIVER MATRIX
  -------------------------------
                 0 1 2 3 4 5 6 7
                 7 0 1 2 3 4 5 6
                 6 7 0 1 2 3 4 5
                 5 6 7 0 1 2 3 4
                 4 5 6 7 0 1 2 3
                 3 4 5 6 7 0 1 2
                 2 3 4 5 6 7 0 1
                 1 2 3 4 5 6 7 0

       RECEIVER MATRIX IS
 -------------------------------
SYMBOL STATE (COLUMNS) / SYMBOL STATE (ROWS)
      7  5  1  3  2  0  4  6
   ---------------------------
  0|  1  3  2  6  7  5  4  0
  7|  0  1  3  2  6  7  5  4
  5|  4  0  1  3  2  6  7  5
  1|  5  4  0  1  3  2  6  7
  3|  7  5  4  0  1  3  2  6
  2|  6  7  5  4  0  1  3  2
  0|  2  6  7  5  4  0  1  3
  4|  3  2  6  7  5  4  0  1
  6|  1  3  2  6  7  5  4  0

          SYMBOL STATE          SYMBOL STATE          DATA STATE
-------------------------------------------------------------------------
  0      0     0     0      0     0     0      0     0     0
-------------------------------------------------------------------------
  1      0     0     0      0     0     1      1     1     1
-------------------------------------------------------------------------
  2      0     0     0      0     1     0      1     0     0
-------------------------------------------------------------------------
  3      0     0     0      0     1     1      1     0     1
-------------------------------------------------------------------------
  4      0     0     0      1     0     0      0     0     1
-------------------------------------------------------------------------
  5      0     0     0      1     0     1      1     1     0
-------------------------------------------------------------------------
  6      0     0     0      1     1     0      0     1     1
-------------------------------------------------------------------------
  7      0     0     0      1     1     1      0     1     0
-------------------------------------------------------------------------
  8      0     0     1      0     0     0      0     1     0
-------------------------------------------------------------------------
  9      0     0     1      0     0     1      0     0     0
-------------------------------------------------------------------------
 10      0     0     1      0     1     0      0     1     1
-------------------------------------------------------------------------
 11      0     0     1      0     1     1      0     0     1
-------------------------------------------------------------------------
 12      0     0     1      1     0     0      1     1     0
-------------------------------------------------------------------------
 13      0     0     1      1     0     1      1     0     0
-------------------------------------------------------------------------
 14      0     0     1      1     1     0      1     1     1
-------------------------------------------------------------------------
 15      0     0     1      1     1     1      1     0     1
```

16	0	1	0	0	0	0	0	0	1
17	0	1	0	0	0	1	1	0	1
18	0	1	0	0	1	0	0	0	0
19	0	1	0	0	1	1	1	0	0
20	0	1	0	1	0	0	0	1	1
21	0	1	0	1	0	1	1	1	1
22	0	1	0	1	1	0	0	1	0
23	0	1	0	1	1	1	1	1	0
24	0	1	1	0	0	0	0	1	1
25	0	1	1	0	0	1	1	0	0
26	0	1	1	0	1	0	0	0	1
27	0	1	1	0	1	1	0	0	0
28	0	1	1	1	0	0	0	1	0
29	0	1	1	1	0	1	1	0	1
30	0	1	1	1	1	0	1	1	0
31	0	1	1	1	1	1	1	1	1
32	1	0	0	0	0	0	1	0	0
33	1	0	0	0	0	1	1	1	0
34	1	0	0	0	1	0	1	0	1
35	1	0	0	0	1	1	1	1	1
36	1	0	0	1	0	0	0	0	0
37	1	0	0	1	0	1	0	1	0
38	1	0	0	1	1	0	0	0	1
39	1	0	0	1	1	1	0	1	1
40	1	0	1	0	0	0	1	1	0
41	1	0	1	0	0	1	0	0	1
42	1	0	1	0	1	0	0	1	0
43	1	0	1	0	1	1	0	1	1
44	1	0	1	1	0	0	1	1	1
45	1	0	1	1	0	1	0	0	0
46	1	0	1	1	1	0	1	0	1
47	1	0	1	1	1	1	1	0	0
48	1	1	0	0	0	0	1	0	1
49	1	1	0	0	0	1	0	1	0
50	1	1	0	0	1	0	1	1	1

```
----------------------------------------------------------------------------
 51     1     1     0        0     1     1        1     1     0
----------------------------------------------------------------------------
 52     1     1     0        1     0     0        1     0     0
----------------------------------------------------------------------------
 53     1     1     0        1     0     1        0     1     1
----------------------------------------------------------------------------
 54     1     1     0        1     1     0        0     0     0
----------------------------------------------------------------------------
 55     1     1     0        1     1     1        0     0     1
----------------------------------------------------------------------------
 56     1     1     1        0     0     0        1     1     1
----------------------------------------------------------------------------
 57     1     1     1        0     0     1        0     1     1
----------------------------------------------------------------------------
 58     1     1     1        0     1     0        1     1     0
----------------------------------------------------------------------------
 59     1     1     1        0     1     1        0     1     0
----------------------------------------------------------------------------
 60     1     1     1        1     0     0        1     0     1
----------------------------------------------------------------------------
 61     1     1     1        1     0     1        0     0     1
----------------------------------------------------------------------------
 62     1     1     1        1     1     0        1     0     0
----------------------------------------------------------------------------
 63     1     1     1        1     1     1        0     0     0
----------------------------------------------------------------------------


                 TRANSMITTER INFORMATION FOR THE GIVEN RECEIVER
----------------------------------------------------------------------------

    GENERAL TRANSMITTER MATRIX
 ------------------------------
 ABCDEFGH
 BCDEFGHA
 CDEFGHAB
 DEFGHABC
 EFGHABCD
 FGHABCDE
 GHABCDEF
 HABCDEFG


    TRANSMITTER MATRIX
------------------------------
SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)
     0  1  3  2  6  7  5  4
-----------------------------------
  7|  7  5  1  3  2  0  4  6
  5|  5  1  3  2  0  4  6  7
  1|  1  3  2  0  4  6  7  5
  3|  3  2  0  4  6  7  5  1
  2|  2  0  4  6  7  5  1  3
  0|  0  4  6  7  5  1  3  2
  4|  4  6  7  5  1  3  2  0
  6|  6  7  5  1  3  2  0  4
```

3. 16-DPSK TRANSMITTER

```
        TRANSMITTER'S ENCODER MATRIX TABLE FOR  8-DPSK
-----------------------------------------------------------------

    GENERAL TRANSMITTER MATRIX
   -----------------------------
 ABCDEFGHIJKLMNOP
 BCDEFGHIJKLMNOPA
 CDEFGHIJKLMNOPAB
 DEFGHIJKLMNOPABC
 EFGHIJKLMNOPABCD
 FGHIJKLMNOPABCDE
 GHIJKLMNOPABCDEF
 HIJKLMNOPABCDEFG
 IJKLMNOPABCDEFGH
 JKLMNOPABCDEFGHI
 KLMNOPABCDEFGHIJ
 LMNOPABCDEFGHIJK
 MNOPABCDEFGHIJKL
 NOPABCDEFGHIJKLM
 OPABCDEFGHIJKLMN
 PABCDEFGHIJKLMNO


    TRANSMITTER MATRIX
-----------------------------

    SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)
      0  1  3  2  6  7  5  4 12 13 15 14 10 11  9  8
----------------------------------------------------------------------
 15| 15 13  9 11  3  1  5  7  6  4  0  2 10  8 12 14
 13| 13  9 11  3  1  5  7  6  4  0  2 10  8 12 14 15
  9|  9 11  3  1  5  7  6  4  0  2 10  8 12 14 15 13
 11| 11  3  1  5  7  6  4  0  2 10  8 12 14 15 13  9
  3|  3  1  5  7  6  4  0  2 10  8 12 14 15 13  9 11
  1|  1  5  7  6  4  0  2 10  8 12 14 15 13  9 11  3
  5|  5  7  6  4  0  2 10  8 12 14 15 13  9 11  3  1
  7|  7  6  4  0  2 10  8 12 14 15 13  9 11  3  1  5
  6|  6  4  0  2 10  8 12 14 15 13  9 11  3  1  5  7
  4|  4  0  2 10  8 12 14 15 13  9 11  3  1  5  7  6
  0|  0  2 10  8 12 14 15 13  9 11  3  1  5  7  6  4
  2|  2 10  8 12 14 15 13  9 11  3  1  5  7  6  4  0
 10| 10  8 12 14 15 13  9 11  3  1  5  7  6  4  0  2
  8|  8 12 14 15 13  9 11  3  1  5  7  6  4  0  2 10
 12| 12 14 15 13  9 11  3  1  5  7  6  4  0  2 10  8
 14| 14 15 13  9 11  3  1  5  7  6  4  0  2 10  8 12

           SYMBOL STATE               DATA STATE              SYMBOL
------------------------------------------------------------------------
  0     0    0    0    0       0    0    0    0       0    0    0    0
------------------------------------------------------------------------
  1     0    0    0    0       0    0    0    1       0    0    1    0
------------------------------------------------------------------------
  2     0    0    0    0       0    0    1    0       1    0    0    0
------------------------------------------------------------------------
  3     0    0    0    0       0    0    1    1       1    0    1    0
------------------------------------------------------------------------
  4     0    0    0    0       0    1    0    0       1    1    0    1
------------------------------------------------------------------------
  5     0    0    0    0       0    1    0    1       1    1    1    1
------------------------------------------------------------------------
  6     0    0    0    0       0    1    1    0       1    1    0    0
------------------------------------------------------------------------
  7     0    0    0    0       0    1    1    1       1    1    1    0
------------------------------------------------------------------------
  8     0    0    0    0       1    0    0    0       0    1    0    0
```

9	0	0	0	0	1	0	0	1	0	1	1	0
10	0	0	0	0	1	0	1	0	0	1	0	1
11	0	0	0	0	1	0	1	1	0	1	1	1
12	0	0	0	0	1	1	0	0	1	0	0	1
13	0	0	0	0	1	1	0	1	1	0	1	1
14	0	0	0	0	1	1	1	0	0	0	0	1
15	0	0	0	0	1	1	1	1	0	0	1	1
16	0	0	0	1	0	0	0	0	0	0	0	1
17	0	0	0	1	0	0	0	1	0	1	0	1
18	0	0	0	1	0	0	1	0	0	1	1	0
19	0	0	0	1	0	0	1	1	0	1	1	1
20	0	0	0	1	0	1	0	0	1	0	1	0
21	0	0	0	1	0	1	0	1	0	0	1	0
22	0	0	0	1	0	1	1	0	0	1	0	0
23	0	0	0	1	0	1	1	1	0	0	0	0
24	0	0	0	1	1	0	0	0	0	0	1	1
25	0	0	0	1	1	0	0	1	1	0	1	1
26	0	0	0	1	1	0	1	0	1	1	0	1
27	0	0	0	1	1	0	1	1	1	0	0	1
28	0	0	0	1	1	1	0	0	1	0	0	0
29	0	0	0	1	1	1	0	1	1	1	0	0
30	0	0	0	1	1	1	1	0	1	1	1	1
31	0	0	0	1	1	1	1	1	1	1	1	0
32	0	0	1	0	0	0	0	0	0	0	1	0
33	0	0	1	0	0	0	0	1	1	0	1	0
34	0	0	1	0	0	0	1	0	1	1	0	0
35	0	0	1	0	0	0	1	1	1	0	0	0
36	0	0	1	0	0	1	0	0	1	0	0	1
37	0	0	1	0	0	1	0	1	1	1	0	1
38	0	0	1	0	0	1	1	0	1	1	1	0
39	0	0	1	0	0	1	1	1	1	1	1	1
40	0	0	1	0	1	0	0	0	0	0	0	0
41	0	0	1	0	1	0	0	1	0	1	0	0
42	0	0	1	0	1	0	1	0	0	1	1	1
43	0	0	1	0	1	0	1	1	0	1	1	0

44	0	0	1	0	1	1	0	0	1	0	1	1
45	0	0	1	0	1	1	0	1	0	0	1	1
46	0	0	1	0	1	1	1	0	0	1	0	1
47	0	0	1	0	1	1	1	1	0	0	0	1
48	0	0	1	1	0	0	0	0	0	0	1	1
49	0	0	1	1	0	0	0	1	0	0	0	1
50	0	0	1	1	0	0	1	0	0	1	1	1
51	0	0	1	1	0	0	1	1	0	1	0	1
52	0	0	1	1	0	1	0	0	0	0	1	0
53	0	0	1	1	0	1	0	1	0	0	0	0
54	0	0	1	1	0	1	1	0	0	1	1	0
55	0	0	1	1	0	1	1	1	0	1	0	0
56	0	0	1	1	1	0	0	0	1	0	1	1
57	0	0	1	1	1	0	0	1	1	0	0	1
58	0	0	1	1	1	0	1	0	1	1	1	1
59	0	0	1	1	1	0	1	1	1	1	0	1
60	0	0	1	1	1	1	0	0	1	0	1	0
61	0	0	1	1	1	1	0	1	1	0	0	0
62	0	0	1	1	1	1	1	0	1	1	1	0
63	0	0	1	1	1	1	1	1	1	1	0	0
64	0	1	0	0	0	0	0	0	0	1	0	0
65	0	1	0	0	0	0	0	1	0	0	0	0
66	0	1	0	0	0	0	1	0	1	0	1	0
67	0	1	0	0	0	0	1	1	0	0	1	0
68	0	1	0	0	0	1	0	0	1	1	1	1
69	0	1	0	0	0	1	0	1	1	1	1	0
70	0	1	0	0	0	1	1	0	1	0	0	0
71	0	1	0	0	0	1	1	1	1	1	0	0
72	0	1	0	0	1	0	0	0	0	1	1	0
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74	0	1	0	0	1	0	1	0	0	0	0	1
75	0	1	0	0	1	0	1	1	0	1	0	1
76	0	1	0	0	1	1	0	0	1	1	0	1
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79	0	1	0	0	1	1	1	1	1	0	1	1
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82	0	1	0	1	0	0	1	0	0	1	0	0
83	0	1	0	1	0	0	1	1	0	1	1	0
84	0	1	0	1	0	1	0	0	1	0	0	0
85	0	1	0	1	0	1	0	1	1	0	1	0
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91	0	1	0	1	1	0	1	1	1	0	1	1
92	0	1	0	1	1	1	0	0	1	1	0	0
93	0	1	0	1	1	1	0	1	1	1	1	0
94	0	1	0	1	1	1	1	0	1	1	0	1
95	0	1	0	1	1	1	1	1	1	1	1	1
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123	0	1	1	1	1	0	1	1	0	0	1	1
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152	1	0	0	1	1	0	0	0	1	1	0	1
153	1	0	0	1	1	0	0	1	1	1	1	1
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157	1	0	0	1	1	1	0	1	0	0	1	0
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166	1	0	1	0	0	1	1	0	1	1	1	1
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168	1	0	1	0	1	0	0	0	0	0	1	0
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176	1	0	1	1	0	0	0	0	1	0	1	1
177	1	0	1	1	0	0	0	1	0	0	1	1
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179	1	0	1	1	0	0	1	1	0	0	0	1
180	1	0	1	1	0	1	0	0	0	0	0	0
181	1	0	1	1	0	1	0	1	0	1	0	0
182	1	0	1	1	0	1	1	0	0	1	1	1
183	1	0	1	1	0	1	1	1	0	1	1	0

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190	1	0	1	1	1	1	1	0	1	1	0	0
191	1	0	1	1	1	1	1	1	1	0	0	0
192	1	1	0	0	0	0	0	0	1	1	0	0
193	1	1	0	0	0	0	0	1	1	1	1	0
194	1	1	0	0	0	0	1	0	1	1	0	1
195	1	1	0	0	0	0	1	1	1	1	1	1
196	1	1	0	0	0	1	0	0	0	0	0	1
197	1	1	0	0	0	1	0	1	0	0	1	1
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199	1	1	0	0	0	1	1	1	1	0	1	1
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203	1	1	0	0	1	0	1	1	0	0	1	0
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206	1	1	0	0	1	1	1	0	0	1	0	0
207	1	1	0	0	1	1	1	1	0	1	1	0
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209	1	1	0	1	0	0	0	1	1	0	0	1
210	1	1	0	1	0	0	1	0	0	0	1	1
211	1	1	0	1	0	0	1	1	1	0	1	1
212	1	1	0	1	0	1	0	0	0	1	1	0
213	1	1	0	1	0	1	0	1	0	1	1	1
214	1	1	0	1	0	1	1	0	0	0	0	1
215	1	1	0	1	0	1	1	1	0	1	0	1
216	1	1	0	1	1	0	0	0	1	1	1	1
217	1	1	0	1	1	0	0	1	1	1	1	0
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219	1	1	0	1	1	0	1	1	1	1	0	0
220	1	1	0	1	1	1	0	0	0	1	0	0
221	1	1	0	1	1	1	0	1	0	0	0	0
222	1	1	0	1	1	1	1	0	1	0	1	0
223	1	1	0	1	1	1	1	1	0	0	1	0
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226	1	1	1	0	0	0	1	0	1	0	0	1
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229	1	1	1	0	0	1	0	1	0	0	0	1
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231	1	1	1	0	0	1	1	1	0	0	1	1
232	1	1	1	0	1	0	0	0	1	1	0	0
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234	1	1	1	0	1	0	1	0	0	0	1	0
235	1	1	1	0	1	0	1	1	1	0	1	0
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238	1	1	1	0	1	1	1	0	0	0	0	0
239	1	1	1	0	1	1	1	1	0	1	0	0
240	1	1	1	1	0	0	0	0	1	1	1	1
241	1	1	1	1	0	0	0	1	1	1	0	1
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243	1	1	1	1	0	0	1	1	1	0	0	1
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245	1	1	1	1	0	1	0	1	0	1	0	1
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247	1	1	1	1	0	1	1	1	0	0	0	1
248	1	1	1	1	1	0	0	0	1	1	1	0
249	1	1	1	1	1	0	0	1	1	1	0	0
250	1	1	1	1	1	0	1	0	1	0	1	0
251	1	1	1	1	1	0	1	1	1	0	0	0
252	1	1	1	1	1	1	0	0	0	1	1	0
253	1	1	1	1	1	1	0	1	0	1	0	0

254	1	1	1	1	1	1	1	0	0	0	1	0
255	1	1	1	1	1	1	1	1	0	0	0	0

4. 16-DPSK RECEIVER

```
          RECEIVER'S DECODER MATRIX TABLE FOR  16-DPSK
-----------------------------------------------------------------

                 GENERAL RECEIVER MATRIX
            -------------------------------
             0 1 2 3 4 5 6 7 8 9101112131415
            15 0 1 2 3 4 5 6 7 8 91011121314
            1415 0 1 2 3 4 5 6 7 8 910111213
            131415 0 1 2 3 4 5 6 7 8 9101112
            12131415 0 1 2 3 4 5 6 7 8 91011
            1112131415 0 1 2 3 4 5 6 7 8 910
            101112131415 0 1 2 3 4 5 6 7 8 9
             9101112131415 0 1 2 3 4 5 6 7 8
             8 9101112131415 0 1 2 3 4 5 6 7
             7 8 9101112131415 0 1 2 3 4 5 6
             6 7 8 9101112131415 0 1 2 3 4 5
             5 6 7 8 9101112131415 0 1 2 3 4
             4 5 6 7 8 9101112131415 0 1 2 3
             3 4 5 6 7 8 9101112131415 0 1 2
             2 3 4 5 6 7 8 9101112131415 0 1
             1 2 3 4 5 6 7 8 9101112131415 0


        RECEIVER MATRIX IS
      ------------------------------

 SYMBOL STATE (COLUMNS) / SYMBOL STATE (ROWS)
     15 13  9 11  3  1  5  7  6  4  0  2 10  8 12 14
--------------------------------------------------------
 15|  0  1  3  2  6  7  5  4 12 13 15 14 10 11  9  8
 13|  8  0  1  3  2  6  7  5  4 12 13 15 14 10 11  9
  9|  9  8  0  1  3  2  6  7  5  4 12 13 15 14 10 11
 11| 11  9  8  0  1  3  2  6  7  5  4 12 13 15 14 10
  3| 10 11  9  8  0  1  3  2  6  7  5  4 12 13 15 14
  1| 14 10 11  9  8  0  1  3  2  6  7  5  4 12 13 15
  5| 15 14 10 11  9  8  0  1  3  2  6  7  5  4 12 13
  7| 13 15 14 10 11  9  8  0  1  3  2  6  7  5  4 12
  6| 12 13 15 14 10 11  9  8  0  1  3  2  6  7  5  4
  4|  4 12 13 15 14 10 11  9  8  0  1  3  2  6  7  5
  0|  5  4 12 13 15 14 10 11  9  8  0  1  3  2  6  7
  2|  7  5  4 12 13 15 14 10 11  9  8  0  1  3  2  6
 10|  6  7  5  4 12 13 15 14 10 11  9  8  0  1  3  2
  8|  2  6  7  5  4 12 13 15 14 10 11  9  8  0  1  3
 12|  3  2  6  7  5  4 12 13 15 14 10 11  9  8  0  1
 14|  1  3  2  6  7  5  4 12 13 15 14 10 11  9  8  0

           SYMBOL STATE               SYMBOL STATE            DATA STATE
------------------------------------------------------------------------
  0     0     0     0     0       0     0     0     0      0     0     0     0
------------------------------------------------------------------------
  1     0     0     0     0       0     0     0     1      1     1     1     0
------------------------------------------------------------------------
  2     0     0     0     0       0     0     1     0      0     0     0     1
------------------------------------------------------------------------
  3     0     0     0     0       0     0     1     1      1     1     1     1
------------------------------------------------------------------------
  4     0     0     0     0       0     1     0     0      1     0     0     0
------------------------------------------------------------------------
  5     0     0     0     0       0     1     0     1      1     0     1     0
------------------------------------------------------------------------
  6     0     0     0     0       0     1     1     0      1     0     0     1
------------------------------------------------------------------------
  7     0     0     0     0       0     1     1     1      1     0     1     1
------------------------------------------------------------------------
```

8	0	0	0	0	1	0	0	0	0	0	1	0
9	0	0	0	0	1	0	0	1	1	1	0	0
10	0	0	0	0	1	0	1	0	0	0	1	1
11	0	0	0	0	1	0	1	1	1	1	0	1
12	0	0	0	0	1	1	0	0	0	1	1	0
13	0	0	0	0	1	1	0	1	0	1	0	0
14	0	0	0	0	1	1	1	0	0	1	1	1
15	0	0	0	0	1	1	1	1	0	1	0	1
16	0	0	0	1	0	0	0	0	0	1	1	1
17	0	0	0	1	0	0	0	1	0	0	0	0
18	0	0	0	1	0	0	1	0	0	1	0	1
19	0	0	0	1	0	0	1	1	1	0	0	0
20	0	0	0	1	0	1	0	0	0	1	1	0
21	0	0	0	1	0	1	0	1	0	0	0	1
22	0	0	0	1	0	1	1	0	0	0	1	0
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24	0	0	0	1	1	0	0	0	1	1	0	0
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33	0	0	1	0	0	0	0	1	1	1	1	1
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37	0	0	1	0	0	1	0	1	1	1	1	0
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48	0	0	1	1	0	0	0	0	0	1	0	1
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65	0	1	0	0	0	0	0	1	1	0	1	0
66	0	1	0	0	0	0	1	0	0	0	1	1
67	0	1	0	0	0	0	1	1	1	1	1	0
68	0	1	0	0	0	1	0	0	0	0	0	0
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71	0	1	0	0	0	1	1	1	1	0	0	1
72	0	1	0	0	1	0	0	0	0	1	1	0
73	0	1	0	0	1	0	0	1	1	1	0	1
74	0	1	0	0	1	0	1	0	0	0	1	0
75	0	1	0	0	1	0	1	1	1	1	1	1
76	0	1	0	0	1	1	0	0	0	1	1	1
77	0	1	0	0	1	1	0	1	1	1	0	0

78	0	1	0	0	1	1	1	0	0	1	0	1
79	0	1	0	0	1	1	1	1	0	1	0	0
80	0	1	0	1	0	0	0	0	0	1	1	0
81	0	1	0	1	0	0	0	1	1	0	0	0
82	0	1	0	1	0	0	1	0	0	1	1	1
83	0	1	0	1	0	0	1	1	1	0	0	1
84	0	1	0	1	0	1	0	0	0	0	1	0
85	0	1	0	1	0	1	0	1	0	0	0	0
86	0	1	0	1	0	1	1	0	0	0	1	1
87	0	1	0	1	0	1	1	1	0	0	0	1
88	0	1	0	1	1	0	0	0	0	1	0	0
89	0	1	0	1	1	0	0	1	1	0	1	0
90	0	1	0	1	1	0	1	0	0	1	0	1
91	0	1	0	1	1	0	1	1	1	0	1	1
92	0	1	0	1	1	1	0	0	1	1	0	0
93	0	1	0	1	1	1	0	1	1	1	1	0
94	0	1	0	1	1	1	1	0	1	1	0	1
95	0	1	0	1	1	1	1	1	1	1	1	1
96	0	1	1	0	0	0	0	0	0	0	1	1
97	0	1	1	0	0	0	0	1	1	0	1	1
98	0	1	1	0	0	0	1	0	0	0	1	0
99	0	1	1	0	0	0	1	1	1	0	1	0
100	0	1	1	0	0	1	0	0	0	0	0	1
101	0	1	1	0	0	1	0	1	1	0	0	1
102	0	1	1	0	0	1	1	0	0	0	0	0
103	0	1	1	0	0	1	1	1	1	0	0	0
104	0	1	1	0	1	0	0	0	0	1	1	1
105	0	1	1	0	1	0	0	1	1	1	1	1
106	0	1	1	0	1	0	1	0	0	1	1	0
107	0	1	1	0	1	0	1	1	1	1	1	0
108	0	1	1	0	1	1	0	0	0	1	0	1
109	0	1	1	0	1	1	0	1	1	1	0	1
110	0	1	1	0	1	1	1	0	0	1	0	0
111	0	1	1	0	1	1	1	1	1	1	0	0
112	0	1	1	1	0	0	0	0	0	0	1	0

113	0	1	1	1	0	0	0	1	1	0	0	1
114	0	1	1	1	0	0	1	0	0	1	1	0
115	0	1	1	1	0	0	1	1	1	0	1	1
116	0	1	1	1	0	1	0	0	0	0	1	1
117	0	1	1	1	0	1	0	1	1	0	0	0
118	0	1	1	1	0	1	1	0	0	0	0	1
119	0	1	1	1	0	1	1	1	0	0	0	0
120	0	1	1	1	1	0	0	0	0	1	0	1
121	0	1	1	1	1	0	0	1	1	1	1	0
122	0	1	1	1	1	0	1	0	0	1	1	1
123	0	1	1	1	1	0	1	1	1	0	1	0
124	0	1	1	1	1	1	0	0	0	1	0	0
125	0	1	1	1	1	1	0	1	1	1	1	1
126	0	1	1	1	1	1	1	0	1	1	0	0
127	0	1	1	1	1	1	1	1	1	1	0	1
128	1	0	0	0	0	0	0	0	1	0	1	1
129	1	0	0	0	0	0	0	1	1	1	0	0
130	1	0	0	0	0	0	1	0	1	0	0	1
131	1	0	0	0	0	0	1	1	0	1	0	0
132	1	0	0	0	0	1	0	0	1	0	1	0
133	1	0	0	0	0	1	0	1	1	1	0	1
134	1	0	0	0	0	1	1	0	1	1	1	0
135	1	0	0	0	0	1	1	1	1	1	1	1
136	1	0	0	0	1	0	0	0	0	0	0	0
137	1	0	0	0	1	0	0	1	0	1	1	1
138	1	0	0	0	1	0	1	0	1	0	0	0
139	1	0	0	0	1	0	1	1	0	1	0	1
140	1	0	0	0	1	1	0	0	0	0	0	1
141	1	0	0	0	1	1	0	1	0	1	1	0
142	1	0	0	0	1	1	1	0	0	0	1	1
143	1	0	0	0	1	1	1	1	0	0	1	0
144	1	0	0	1	0	0	0	0	1	1	0	0
145	1	0	0	1	0	0	0	1	0	0	1	0
146	1	0	0	1	0	0	1	0	1	1	0	1
147	1	0	0	1	0	0	1	1	0	0	1	1

148	1	0	0	1	0	1	0	0	0	1	0	0
149	1	0	0	1	0	1	0	1	0	1	1	0
150	1	0	0	1	0	1	1	0	0	1	0	1
151	1	0	0	1	0	1	1	1	0	1	1	1
152	1	0	0	1	1	0	0	0	1	1	1	0
153	1	0	0	1	1	0	0	1	0	0	0	0
154	1	0	0	1	1	0	1	0	1	1	1	1
155	1	0	0	1	1	0	1	1	0	0	0	1
156	1	0	0	1	1	1	0	0	1	0	1	0
157	1	0	0	1	1	1	0	1	1	0	0	0
158	1	0	0	1	1	1	1	0	1	0	1	1
159	1	0	0	1	1	1	1	1	1	0	0	1
160	1	0	1	0	0	0	0	0	1	0	0	1
161	1	0	1	0	0	0	0	1	1	1	0	1
162	1	0	1	0	0	0	1	0	1	0	0	0
163	1	0	1	0	0	0	1	1	1	1	0	0
164	1	0	1	0	0	1	0	0	1	0	1	1
165	1	0	1	0	0	1	0	1	1	1	1	1
166	1	0	1	0	0	1	1	0	1	0	1	0
167	1	0	1	0	0	1	1	1	1	1	1	0
168	1	0	1	0	1	0	0	0	0	0	0	1
169	1	0	1	0	1	0	0	1	0	1	0	1
170	1	0	1	0	1	0	1	0	0	0	0	0
171	1	0	1	0	1	0	1	1	0	1	0	0
172	1	0	1	0	1	1	0	0	0	0	1	1
173	1	0	1	0	1	1	0	1	0	1	1	1
174	1	0	1	0	1	1	1	0	0	0	1	0
175	1	0	1	0	1	1	1	1	0	1	1	0
176	1	0	1	1	0	0	0	0	0	1	0	0
177	1	0	1	1	0	0	0	1	0	0	1	1
178	1	0	1	1	0	0	1	0	1	1	0	0
179	1	0	1	1	0	0	1	1	0	0	0	1
180	1	0	1	1	0	1	0	0	0	1	0	1
181	1	0	1	1	0	1	0	1	0	0	1	0
182	1	0	1	1	0	1	1	0	0	1	1	1

183	1	0	1	1	0	1	1	1	0	1	1	0
184	1	0	1	1	1	0	0	0	1	1	1	1
185	1	0	1	1	1	0	0	1	1	0	0	0
186	1	0	1	1	1	0	1	0	1	1	0	1
187	1	0	1	1	1	0	1	1	0	0	0	0
188	1	0	1	1	1	1	0	0	1	1	1	0
189	1	0	1	1	1	1	0	1	1	0	0	1
190	1	0	1	1	1	1	1	0	1	0	1	0
191	1	0	1	1	1	1	1	1	1	0	1	1
192	1	1	0	0	0	0	0	0	1	0	1	0
193	1	1	0	0	0	0	0	1	0	1	0	0
194	1	1	0	0	0	0	1	0	1	0	1	1
195	1	1	0	0	0	0	1	1	0	1	0	1
196	1	1	0	0	0	1	0	0	1	1	1	0
197	1	1	0	0	0	1	0	1	1	1	0	0
198	1	1	0	0	0	1	1	0	1	1	1	1
199	1	1	0	0	0	1	1	1	1	1	0	1
200	1	1	0	0	1	0	0	0	1	0	0	0
201	1	1	0	0	1	0	0	1	0	1	1	0
202	1	1	0	0	1	0	1	0	1	0	0	1
203	1	1	0	0	1	0	1	1	0	1	1	1
204	1	1	0	0	1	1	0	0	0	0	0	0
205	1	1	0	0	1	1	0	1	0	0	1	0
206	1	1	0	0	1	1	1	0	0	0	0	1
207	1	1	0	0	1	1	1	1	0	0	1	1
208	1	1	0	1	0	0	0	0	1	1	0	1
209	1	1	0	1	0	0	0	1	0	1	1	0
210	1	1	0	1	0	0	1	0	1	1	1	1
211	1	1	0	1	0	0	1	1	0	0	1	0
212	1	1	0	1	0	1	0	0	1	1	0	0
213	1	1	0	1	0	1	0	1	0	1	1	1
214	1	1	0	1	0	1	1	0	0	1	0	0
215	1	1	0	1	0	1	1	1	0	1	0	1
216	1	1	0	1	1	0	0	0	1	0	1	0
217	1	1	0	1	1	0	0	1	0	0	0	1

218	1	1	0	1	1	0	1	0	1	1	1	0
219	1	1	0	1	1	0	1	1	0	0	1	1
220	1	1	0	1	1	1	0	0	1	0	1	1
221	1	1	0	1	1	1	0	1	0	0	0	0
222	1	1	0	1	1	1	1	0	1	0	0	1
223	1	1	0	1	1	1	1	1	1	0	0	0
224	1	1	1	0	0	0	0	0	1	1	1	0
225	1	1	1	0	0	0	0	1	0	1	0	1
226	1	1	1	0	0	0	1	0	1	0	1	0
227	1	1	1	0	0	0	1	1	0	1	1	1
228	1	1	1	0	0	1	0	0	1	1	1	1
229	1	1	1	0	0	1	0	1	0	1	0	0
230	1	1	1	0	0	1	1	0	1	1	0	1
231	1	1	1	0	0	1	1	1	1	1	0	0
232	1	1	1	0	1	0	0	0	1	0	0	1
233	1	1	1	0	1	0	0	1	0	0	1	0
234	1	1	1	0	1	0	1	0	1	0	1	1
235	1	1	1	0	1	0	1	1	0	1	1	0
236	1	1	1	0	1	1	0	0	1	0	0	0
237	1	1	1	0	1	1	0	1	0	0	1	1
238	1	1	1	0	1	1	1	0	0	0	0	0
239	1	1	1	0	1	1	1	1	0	0	0	1
240	1	1	1	1	0	0	0	0	1	1	1	1
241	1	1	1	1	0	0	0	1	0	1	1	1
242	1	1	1	1	0	0	1	0	1	1	1	0
243	1	1	1	1	0	0	1	1	0	1	1	0
244	1	1	1	1	0	1	0	0	1	1	0	1
245	1	1	1	1	0	1	0	1	0	1	0	1
246	1	1	1	1	0	1	1	0	1	1	0	0
247	1	1	1	1	0	1	1	1	0	1	0	0
248	1	1	1	1	1	0	0	0	1	0	1	1
249	1	1	1	1	1	0	0	1	0	0	1	1
250	1	1	1	1	1	0	1	0	1	0	1	0
251	1	1	1	1	1	0	1	1	0	0	1	0
252	1	1	1	1	1	1	0	0	1	0	0	1

```
253     1   1   1   1      1   1   0   1      0   0   0   1
-------------------------------------------------------------
254     1   1   1   1      1   1   1   0      1   0   0   0
-------------------------------------------------------------
255     1   1   1   1      1   1   1   1      0   0   0   0
-------------------------------------------------------------


              TRANSMITTER INFORMATION FOR THE GIVEN RECEIVER
-------------------------------------------------------------

    GENERAL TRANSMITTER MATRIX
 ------------------------------
 ABCDEFGHIJKLMNOP
 BCDEFGHIJKLMNOPA
 CDEFGHIJKLMNOPAB
 DEFGHIJKLMNOPABC
 EFGHIJKLMNOPABCD
 FGHIJKLMNOPABCDE
 GHIJKLMNOPABCDEF
 HIJKLMNOPABCDEFG
 IJKLMNOPABCDEFGH
 JKLMNOPABCDEFGHI
 KLMNOPABCDEFGHIJ
 LMNOPABCDEFGHIJK
 MNOPABCDEFGHIJKL
 NOPABCDEFGHIJKLM
 OPABCDEFGHIJKLMN
 PABCDEFGHIJKLMNO


    TRANSMITTER MATRIX
------------------------------

SYMBOL STATE (COLUMNS) / DATA STATE (ROWS)
      0  1  3  2  6  7  5  4 12 13 15 14 10 11  9  8
 ----------------------------------------------------------------
 15| 15 13  9 11  3  1  5  7  6  4  0  2 10  8 12 14
 13| 13  9 11  3  1  5  7  6  4  0  2 10  8 12 14 15
  9|  9 11  3  1  5  7  6  4  0  2 10  8 12 14 15 13
 11| 11  3  1  5  7  6  4  0  2 10  8 12 14 15 13  9
  3|  3  1  5  7  6  4  0  2 10  8 12 14 15 13  9 11
  1|  1  5  7  6  4  0  2 10  8 12 14 15 13  9 11  3
  5|  5  7  6  4  0  2 10  8 12 14 15 13  9 11  3  1
  7|  7  6  4  0  2 10  8 12 14 15 13  9 11  3  1  5
  6|  6  4  0  2 10  8 12 14 15 13  9 11  3  1  5  7
  4|  4  0  2 10  8 12 14 15 13  9 11  3  1  5  7  6
  0|  0  2 10  8 12 14 15 13  9 11  3  1  5  7  6  4
  2|  2 10  8 12 14 15 13  9 11  3  1  5  7  6  4  0
 10| 10  8 12 14 15 13  9 11  3  1  5  7  6  4  0  2
  8|  8 12 14 15 13  9 11  3  1  5  7  6  4  0  2 10
 12| 12 14 15 13  9 11  3  1  5  7  6  4  0  2 10  8
 14| 14 15 13  9 11  3  1  5  7  6  4  0  2 10  8 12
```

APPENDIX C

COMPUTER GENERATED OUTPUTS FOR DBDR'S ERROR PATTERN ANALYSES

The program outputs given below are a partial sample of the main outputs. Since undetectable error patterns are similar, in the next sections, 1/8 period of the pattern have been included. Actually 526 combinations of error patterns are detected by the program for the 8-DPSK case. For the 16-DPSK case there are 4096 undetectable error pattern combinations.

1. 8-DPSK RECEIVER

```
######################################################################
       CORRECT RECEPTION COMBINATIONS IN PRESENCE OF ERROR
######################################################################

 INPUT     PREVIOUS     PRESENT      PREVIOUS    PRESENT      OUTPUT
         TRANSMISSION TRANSMISSION  RECEPTION   RECEPTION
----------------------------------------------------------------------
0  0  0     0  0  0      0  0  0       0  0  0     0  0  0    0  0  0
                                       0  0  1     0  0  1    0  0  0
                                       0  1  0     0  1  0    0  0  0
                                       0  1  1     0  1  1    0  0  0
                                       1  0  0     1  0  0    0  0  0
                                       1  0  1     1  0  1    0  0  0
                                       1  1  0     1  1  0    0  0  0
                                       1  1  1     1  1  1    0  0  0
----------------------------------------------------------------------
0  0  1     0  0  0      1  0  0       0  0  0     1  0  0    0  0  1
                                       0  0  1     0  1  1    0  0  1
                                       0  1  0     0  0  0    0  0  1
                                       0  1  1     0  1  0    0  0  1
                                       1  0  0     1  1  0    0  0  1
                                       1  0  1     0  0  1    0  0  1
                                       1  1  0     1  1  1    0  0  1
                                       1  1  1     1  0  1    0  0  1
----------------------------------------------------------------------
0  1  0     0  0  0      1  1  1       0  0  0     1  1  1    0  1  0
                                       0  0  1     0  0  0    0  1  0
                                       0  1  0     1  1  0    0  1  0
                                       0  1  1     1  0  0    0  1  0
                                       1  0  0     1  0  1    0  1  0
                                       1  0  1     0  1  0    0  1  0
                                       1  1  0     0  0  1    0  1  0
                                       1  1  1     0  1  1    0  1  0
----------------------------------------------------------------------
0  1  1     0  0  0      1  1  0       0  0  0     1  1  0    0  1  1
                                       0  0  1     0  1  0    0  1  1
                                       0  1  0     1  0  0    0  1  1
                                       0  1  1     0  0  0    0  1  1
                                       1  0  0     1  1  1    0  1  1
                                       1  0  1     0  1  1    0  1  1
                                       1  1  0     1  0  1    0  1  1
                                       1  1  1     0  0  1    0  1  1
----------------------------------------------------------------------
```

1	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0
									0	0	1	1	0	1	1	0	0
									0	1	0	0	1	1	1	0	0
									0	1	1	0	0	1	1	0	0
									1	0	0	0	0	0	1	0	0
									1	0	1	1	1	1	1	0	0
									1	1	0	1	0	0	1	0	0
									1	1	1	1	1	0	1	0	0
1	0	1	0	0	0	0	1	1	0	0	0	0	1	1	1	0	1
									0	0	1	1	1	1	1	0	1
									0	1	0	0	0	1	1	0	1
									0	1	1	1	0	1	1	0	1
									1	0	0	0	1	0	1	0	1
									1	0	1	1	1	0	1	0	1
									1	1	0	0	0	0	1	0	1
									1	1	1	1	0	0	1	0	1
1	1	0	0	0	0	1	0	1	0	0	0	1	0	1	1	1	0
									0	0	1	1	0	0	1	1	0
									0	1	0	1	1	1	1	1	0
									0	1	1	1	1	0	1	1	0
									1	0	0	0	0	1	1	1	0
									1	0	1	0	0	0	1	1	0
									1	1	0	0	1	1	1	1	0
									1	1	1	0	1	0	1	1	0
1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	1	1	1
									0	0	1	1	1	0	1	1	1
									0	1	0	1	0	1	1	1	1
									0	1	1	1	1	1	1	1	1
									1	0	0	0	1	1	1	1	1
									1	0	1	1	0	0	1	1	1
									1	1	0	0	1	0	1	1	1
									1	1	1	0	0	0	1	1	1

2. 16-DPSK RECEIVER

```
########################################################################
       CORRECT RECEPTION COMBINATIONS IN PRECENSE OF ERROR
########################################################################

    INPUT     PREVIOUS     PRESENT      PREVIOUS    PRESENT      OUTPUT
           TRANSMISSION  TRANSMISSION   RECEPTION   RECEPTION
-------------------------------------------------------------------------
0 0 0 0       0 0 0 0      0 0 0 0       0 0 0 0     0 0 0 0    0 0 0 0
                                         0 0 0 1     0 0 0 1    0 0 0 0
                                         0 0 1 0     0 0 1 0    0 0 0 0
                                         0 0 1 1     0 0 1 1    0 0 0 0
                                         0 1 0 0     0 1 0 0    0 0 0 0
                                         0 1 0 1     0 1 0 1    0 0 0 0
                                         0 1 1 0     0 1 1 0    0 0 0 0
                                         0 1 1 1     0 1 1 1    0 0 0 0
                                         1 0 0 0     1 0 0 0    0 0 0 0
                                         1 0 0 1     1 0 0 1    0 0 0 0
                                         1 0 1 0     1 0 1 0    0 0 0 0
                                         1 0 1 1     1 0 1 1    0 0 0 0
                                         1 1 0 0     1 1 0 0    0 0 0 0
                                         1 1 0 1     1 1 0 1    0 0 0 0
                                         1 1 1 0     1 1 1 0    0 0 0 0
                                         1 1 1 1     1 1 1 1    0 0 0 0
----------------------------------------------------------------------
0 0 0 1       0 0 0 0      0 0 1 0       0 0 0 0     0 0 1 0    0 0 0 1
                                         0 0 0 1     0 1 0 1    0 0 0 1
                                         0 0 1 0     1 0 1 0    0 0 0 1
                                         0 0 1 1     0 0 0 1    0 0 0 1
                                         0 1 0 0     0 0 0 0    0 0 0 1
                                         0 1 0 1     0 1 1 1    0 0 0 1
                                         0 1 1 0     0 1 0 0    0 0 0 1
                                         0 1 1 1     0 1 1 0    0 0 0 1
                                         1 0 0 0     1 1 0 0    0 0 0 1
                                         1 0 0 1     1 0 1 1    0 0 0 1
                                         1 0 1 0     1 0 0 0    0 0 0 1
                                         1 0 1 1     0 0 1 1    0 0 0 1
                                         1 1 0 0     1 1 1 0    0 0 0 1
                                         1 1 0 1     1 0 0 1    0 0 0 1
                                         1 1 1 0     1 1 1 1    0 0 0 1
                                         1 1 1 1     1 1 0 1    0 0 0 1
----------------------------------------------------------------------
0 0 1 0       0 0 0 0      1 0 0 0       0 0 0 0     1 0 0 0    0 0 1 0
                                         0 0 0 1     0 1 1 0    0 0 1 0
                                         0 0 1 0     1 1 0 0    0 0 1 0
                                         0 0 1 1     0 1 1 1    0 0 1 0
                                         0 1 0 0     1 0 1 0    0 0 1 0
                                         0 1 0 1     0 1 0 0    0 0 1 0
                                         0 1 1 0     0 0 1 0    0 0 1 0
                                         0 1 1 1     0 0 0 0    0 0 1 0
                                         1 0 0 0     1 1 1 1    0 0 1 0
                                         1 0 0 1     0 0 0 1    0 0 1 0
                                         1 0 1 0     1 1 1 0    0 0 1 0
                                         1 0 1 1     0 1 0 1    0 0 1 0
                                         1 1 0 0     1 1 0 1    0 0 1 0
                                         1 1 0 1     0 0 1 1    0 0 1 0
                                         1 1 1 0     1 0 0 1    0 0 1 0
                                         1 1 1 1     1 0 1 1    0 0 1 0
----------------------------------------------------------------------
0 0 1 1       0 0 0 0      1 0 1 0       0 0 0 0     1 0 1 0    0 0 1 1
                                         0 0 0 1     0 1 1 1    0 0 1 1
                                         0 0 1 0     1 0 0 0    0 0 1 1
                                         0 0 1 1     0 1 0 1    0 0 1 1
```

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                                       0 1 0 0    0 0 1 0    0 0 1 1
                                       0 1 0 1    0 1 1 0    0 0 1 1
                                       0 1 1 0    0 0 0 0    0 0 1 1
                                       0 1 1 1    0 1 0 0    0 0 1 1
                                       1 0 0 0    1 1 1 0    0 0 1 1
                                       1 0 0 1    0 0 1 1    0 0 1 1
                                       1 0 1 0    1 1 0 0    0 0 1 1
                                       1 0 1 1    0 0 0 1    0 0 1 1
                                       1 1 0 0    1 1 1 1    0 0 1 1
                                       1 1 0 1    1 0 1 1    0 0 1 1
                                       1 1 1 0    1 1 0 1    0 0 1 1
                                       1 1 1 1    1 0 0 1    0 0 1 1
-----------------------------------------------------------------------
0 1 0 0      0 0 0 0      1 1 0 1      0 0 0 0    1 1 0 1    0 1 0 0
                                       0 0 0 1    1 0 1 0    0 1 0 0
                                       0 0 1 0    1 0 0 1    0 1 0 0
                                       0 0 1 1    0 0 1 0    0 1 0 0
                                       0 1 0 0    1 1 1 1    0 1 0 0
                                       0 1 0 1    1 0 0 0    0 1 0 0
                                       0 1 1 0    1 1 1 0    0 1 0 0
                                       0 1 1 1    1 1 0 0    0 1 0 0
                                       1 0 0 0    0 0 1 1    0 1 0 0
                                       1 0 0 1    0 1 0 0    0 1 0 0
                                       1 0 1 0    1 0 1 1    0 1 0 0
                                       1 0 1 1    0 0 0 0    0 1 0 0
                                       1 1 0 0    0 0 0 1    0 1 0 0
                                       1 1 0 1    0 1 1 0    0 1 0 0
                                       1 1 1 0    0 1 0 1    0 1 0 0
                                       1 1 1 1    0 1 1 1    0 1 0 0
-----------------------------------------------------------------------
0 1 0 1      0 0 0 0      1 1 1 1      0 0 0 0    1 1 1 1    0 1 0 1
                                       0 0 0 1    0 0 1 0    0 1 0 1
                                       0 0 1 0    1 1 0 1    0 1 0 1
                                       0 0 1 1    0 0 0 0    0 1 0 1
                                       0 1 0 0    1 1 1 0    0 1 0 1
                                       0 1 0 1    1 0 1 0    0 1 0 1
                                       0 1 1 0    1 1 0 0    0 1 0 1
                                       0 1 1 1    1 0 0 0    0 1 0 1
                                       1 0 0 0    1 0 1 1    0 1 0 1
                                       1 0 0 1    0 1 1 0    0 1 0 1
                                       1 0 1 0    1 0 0 1    0 1 0 1
                                       1 0 1 1    0 1 0 0    0 1 0 1
                                       1 1 0 0    0 0 1 1    0 1 0 1
                                       1 1 0 1    0 1 1 1    0 1 0 1
                                       1 1 1 0    0 0 0 1    0 1 0 1
                                       1 1 1 1    0 1 0 1    0 1 0 1
-----------------------------------------------------------------------
0 1 1 0      0 0 0 0      1 1 0 0      0 0 0 0    1 1 0 0    0 1 1 0
                                       0 0 0 1    0 1 0 0    0 1 1 0
                                       0 0 1 0    1 1 1 0    0 1 1 0
                                       0 0 1 1    0 1 1 0    0 1 1 0
                                       0 1 0 0    1 0 0 0    0 1 1 0
                                       0 1 0 1    0 0 0 0    0 1 1 0
                                       0 1 1 0    1 0 1 0    0 1 1 0
                                       0 1 1 1    0 0 1 0    0 1 1 0
                                       1 0 0 0    1 1 0 1    0 1 1 0
                                       1 0 0 1    0 1 0 1    0 1 1 0
                                       1 0 1 0    1 1 1 1    0 1 1 0
                                       1 0 1 1    0 1 1 1    0 1 1 0
                                       1 1 0 0    1 0 0 1    0 1 1 0
                                       1 1 0 1    0 0 0 1    0 1 1 0
                                       1 1 1 0    1 0 1 1    0 1 1 0
                                       1 1 1 1    0 0 1 1    0 1 1 0
-----------------------------------------------------------------------
0 1 1 1      0 0 0 0      1 1 1 0      0 0 0 0    1 1 1 0    0 1 1 1
                                       0 0 0 1    0 0 0 0    0 1 1 1
                                       0 0 1 0    1 1 1 1    0 1 1 1
                                       0 0 1 1    0 1 0 0    0 1 1 1
                                       0 1 0 0    1 1 0 0    0 1 1 1
                                       0 1 0 1    0 0 1 0    0 1 1 1
```

```
                                                0 1 1 0    1 0 0 0    0 1 1 1
                                                0 1 1 1    1 0 1 0    0 1 1 1
                                                1 0 0 0    1 0 0 1    0 1 1 1
                                                1 0 0 1    0 1 1 1    0 1 1 1
                                                1 0 1 0    1 1 0 1    0 1 1 1
                                                1 0 1 1    0 1 1 0    0 1 1 1
                                                1 1 0 0    1 0 1 1    0 1 1 1
                                                1 1 0 1    0 1 0 1    0 1 1 1
                                                1 1 1 0    0 0 1 1    0 1 1 1
                                                1 1 1 1    0 0 0 1    0 1 1 1
-----------------------------------------------------------------------------
1 0 0 0        0 0 0 0        0 1 0 0           0 0 0 0    0 1 0 0    1 0 0 0
                                                0 0 0 1    0 0 1 1    1 0 0 0
                                                0 0 1 0    0 0 0 0    1 0 0 0
                                                0 0 1 1    1 0 1 1    1 0 0 0
                                                0 1 0 0    0 1 1 0    1 0 0 0
                                                0 1 0 1    0 0 0 1    1 0 0 0
                                                0 1 1 0    0 1 1 1    1 0 0 0
                                                0 1 1 1    0 1 0 1    1 0 0 0
                                                1 0 0 0    1 0 1 0    1 0 0 0
                                                1 0 0 1    1 1 0 1    1 0 0 0
                                                1 0 1 0    0 0 1 0    1 0 0 0
                                                1 0 1 1    1 0 0 1    1 0 0 0
                                                1 1 0 0    1 0 0 0    1 0 0 0
                                                1 1 0 1    1 1 1 1    1 0 0 0
                                                1 1 1 0    1 1 0 0    1 0 0 0
                                                1 1 1 1    1 1 1 0    1 0 0 0
-----------------------------------------------------------------------------
1 0 0 1        0 0 0 0        0 1 1 0           0 0 0 0    0 1 1 0    1 0 0 1
                                                0 0 0 1    1 0 1 1    1 0 0 1
                                                0 0 1 0    0 1 0 0    1 0 0 1
                                                0 0 1 1    1 0 0 1    1 0 0 1
                                                0 1 0 0    0 1 1 1    1 0 0 1
                                                0 1 0 1    0 0 1 1    1 0 0 1
                                                0 1 1 0    0 1 0 1    1 0 0 1
                                                0 1 1 1    0 0 0 1    1 0 0 1
                                                1 0 0 0    0 0 1 0    1 0 0 1
                                                1 0 0 1    1 1 1 1    1 0 0 1
                                                1 0 1 0    0 0 0 0    1 0 0 1
                                                1 0 1 1    1 1 0 1    1 0 0 1
                                                1 1 0 0    1 0 1 0    1 0 0 1
                                                1 1 0 1    1 1 1 0    1 0 0 1
                                                1 1 1 0    1 0 0 0    1 0 0 1
                                                1 1 1 1    1 1 0 0    1 0 0 1
-----------------------------------------------------------------------------
1 0 1 0        0 0 0 0        0 1 0 1           0 0 0 0    0 1 0 1    1 0 1 0
                                                0 0 0 1    1 1 0 1    1 0 1 0
                                                0 0 1 0    0 1 1 1    1 0 1 0
                                                0 0 1 1    1 1 1 1    1 0 1 0
                                                0 1 0 0    0 0 0 1    1 0 1 0
                                                0 1 0 1    1 0 0 1    1 0 1 0
                                                0 1 1 0    0 0 1 1    1 0 1 0
                                                0 1 1 1    1 0 1 1    1 0 1 0
                                                1 0 0 0    0 1 0 0    1 0 1 0
                                                1 0 0 1    1 1 0 0    1 0 1 0
                                                1 0 1 0    0 1 1 0    1 0 1 0
                                                1 0 1 1    1 1 1 0    1 0 1 0
                                                1 1 0 0    0 0 0 0    1 0 1 0
                                                1 1 0 1    1 0 0 0    1 0 1 0
                                                1 1 1 0    0 0 1 0    1 0 1 0
                                                1 1 1 1    1 0 1 0    1 0 1 0
-----------------------------------------------------------------------------
1 0 1 1        0 0 0 0        0 1 1 1           0 0 0 0    0 1 1 1    1 0 1 1
                                                0 0 0 1    1 0 0 1    1 0 1 1
                                                0 0 1 0    0 1 1 0    1 0 1 1
                                                0 0 1 1    1 1 0 1    1 0 1 1
                                                0 1 0 0    0 1 0 1    1 0 1 1
                                                0 1 0 1    1 0 1 1    1 0 1 1
                                                0 1 1 0    0 0 0 1    1 0 1 1
                                                0 1 1 1    0 0 1 1    1 0 1 1
```

			1 0 0 0	0 0 0 0	1 0 1 1
			1 0 0 1	1 1 1 0	1 0 1 1
			1 0 1 0	0 1 0 0	1 0 1 1
			1 0 1 1	1 1 1 1	1 0 1 1
			1 1 0 0	0 0 1 0	1 0 1 1
			1 1 0 1	1 1 0 0	1 0 1 1
			1 1 1 0	1 0 1 0	1 0 1 1
			1 1 1 1	1 0 0 0	1 0 1 1
1 1 0 0	0 0 0 0	1 0 0 1	0 0 0 0	1 0 0 1	1 1 0 0
			0 0 0 1	1 0 0 0	1 1 0 0
			0 0 1 0	1 0 1 1	1 1 0 0
			0 0 1 1	1 0 1 0	1 1 0 0
			0 1 0 0	1 1 0 1	1 1 0 0
			0 1 0 1	1 1 0 0	1 1 0 0
			0 1 1 0	1 1 1 1	1 1 0 0
			0 1 1 1	1 1 1 0	1 1 0 0
			1 0 0 0	0 0 0 1	1 1 0 0
			1 0 0 1	0 0 0 0	1 1 0 0
			1 0 1 0	0 0 1 1	1 1 0 0
			1 0 1 1	0 0 1 0	1 1 0 0
			1 1 0 0	0 1 0 1	1 1 0 0
			1 1 0 1	0 1 0 0	1 1 0 0
			1 1 1 0	0 1 1 1	1 1 0 0
			1 1 1 1	0 1 1 0	1 1 0 0
1 1 0 1	0 0 0 0	1 0 1 1	0 0 0 0	1 0 1 1	1 1 0 1
			0 0 0 1	1 1 0 0	1 1 0 1
			0 0 1 0	0 0 1 1	1 1 0 1
			0 0 1 1	1 0 0 0	1 1 0 1
			0 1 0 0	1 0 0 1	1 1 0 1
			0 1 0 1	1 1 1 0	1 1 0 1
			0 1 1 0	1 1 0 1	1 1 0 1
			0 1 1 1	1 1 1 1	1 1 0 1
			1 0 0 0	0 1 0 1	1 1 0 1
			1 0 0 1	0 0 1 0	1 1 0 1
			1 0 1 0	0 0 0 1	1 1 0 1
			1 0 1 1	1 0 1 0	1 1 0 1
			1 1 0 0	0 1 1 1	1 1 0 1
			1 1 0 1	0 0 0 0	1 1 0 1
			1 1 1 0	0 1 1 0	1 1 0 1
			1 1 1 1	0 1 0 0	1 1 0 1
1 1 1 0	0 0 0 0	0 0 0 1	0 0 0 0	0 0 0 1	1 1 1 0
			0 0 0 1	1 1 1 1	1 1 1 0
			0 0 1 0	0 1 0 1	1 1 1 0
			0 0 1 1	1 1 1 0	1 1 1 0
			0 1 0 0	0 0 1 1	1 1 1 0
			0 1 0 1	1 1 0 1	1 1 1 0
			0 1 1 0	1 0 1 1	1 1 1 0
			0 1 1 1	1 0 0 1	1 1 1 0
			1 0 0 0	0 1 1 0	1 1 1 0
			1 0 0 1	1 0 0 0	1 1 1 0
			1 0 1 0	0 1 1 1	1 1 1 0
			1 0 1 1	1 1 0 0	1 1 1 0
			1 1 0 0	0 1 0 0	1 1 1 0
			1 1 0 1	1 0 1 0	1 1 1 0
			1 1 1 0	0 0 0 0	1 1 1 0
			1 1 1 1	0 0 1 0	1 1 1 0
1 1 1 1	0 0 0 0	0 0 1 1	0 0 0 0	0 0 1 1	1 1 1 1
			0 0 0 1	1 1 1 0	1 1 1 1
			0 0 1 0	0 0 0 1	1 1 1 1
			0 0 1 1	1 1 0 0	1 1 1 1
			0 1 0 0	1 0 1 1	1 1 1 1
			0 1 0 1	1 1 1 1	1 1 1 1
			0 1 1 0	1 0 0 1	1 1 1 1
			0 1 1 1	1 1 0 1	1 1 1 1
			1 0 0 0	0 1 1 1	1 1 1 1
			1 0 0 1	1 0 1 0	1 1 1 1

```
1 0 1 0     0 1 0 1     1 1 1 1
1 0 1 1     1 0 0 0     1 1 1 1
1 1 0 0     0 1 1 0     1 1 1 1
1 1 0 1     0 0 1 0     1 1 1 1
1 1 1 0     0 1 0 0     1 1 1 1
1 1 1 1     0 0 0 0     1 1 1 1
```

LIST OF REFERENCES

1. Prabhu, V. K., *On the Performances of Phase-Shift Keying Systems*, The Bell System Technical Journal, V. 60, No. 10, pp. 2307-2343, December 1981.

2. Lindsey, W. C. and M. K. Simon, *Telecommunication Systems Engineering*, Prentice Hall, Englewood Cliffs, New Jersey, 1973 .

3. Ziemer, R. E. and R. L. Peterson, *Digital Communications and Spread Spectrum Systems*, MacMillian, New York, 1985.

4. Lee, P. J., *Computation of the Bit Error Rate of Coherent M-ary PSK with Gray Code Bit Mapping*, IEEE Transactions on Communications, Vol. Com. 34, No. 5 pp.488-491, May 1986.

5. Tan, C. S., *Performance of Direct Bit Detection Receivers for Multiple level Phase-Shift-Keyed Modulation*, Masters Thesis, Naval Postgraduate School, Monterey, California, June 1986.

6. Thompson, G. E., *Experimental Evaluation of a New Form of M-ary* ($M=8$) *Phase-Shift Keying Including Design of the Transmitter and Receiver*, Masters Thesis, Naval Postgraduate School, Monterey, California, December 1984.

7. Myers, G. A. and D. C. Bukofzer, *A Simple Two Channel Receiver for 8-PSK Having Optimum Noise Performance*, Paper submitted to IEEE Transactions on Communications.

8. Bukofzer, D. C., *A Two Channel Direct Bit Detection Receiver for 16-PSK Modulated Signals*, to be published in IEE Proceedings on Communication and Signal Processing.

9. Peebles, P. Z. Jr., *Digital Communication Systems*, Prentice Hall, Englewood Cliffs, New Jersey, 1987.

10. Ha, Tri T., *Digital Satellite Communication*, MacMillian Publishing Company, New York, 1986.

11. Taub H., and D. L. Schilling, *Principles of Communication Systems*, McGraw Hill, New York, 1986.

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